



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR
FOURTH YEAR SECOND SEMESTER
SPECIAL/ SUPPLEMENTARY EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
MATHEMATICS

COURSE CODE: STA 421

COURSE TITLE: MULTIVARIATE ANALYSIS

DATE: 16/11/2022

TIME: 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION 1: (30 Marks) (COMPULSORY)

- a) Define the following terms as used in multivariate statistics. [3mks]
- i. Multiple correlation [3mks]
 - ii. Moment generating function [3mks]
 - iii. Hotelling T^2 distribution [3mks]
 - iv. Principal component [3mks]
- b) Define the coefficient of skewness and Kurtosis for a multivariate vector with a normal distribution i.e. $x \sim N_p(\mu, \Sigma)$ [6mks]
- c) Given that $x \sim N_p(\mu, \Sigma)$, show that $E(x - \mu)(x - \mu)' = \Sigma$ [6mks]
- d) Show that the sample variance-covariance matrix S is given by $S = \frac{1}{n} X'HX$ [6mks]
where $H = I - \frac{1}{n} 11'$

QUESTION 2: (20 Marks)

- a) Show that if $V_1 \sim W_p(\Sigma, n_1)$ and $V_2 \sim W_p(\Sigma, n_2)$ then, $V_1 + V_2 \sim W_p(\Sigma, n_1 + n_2)$ if V_1 and V_2 are independent and hence give the general $V_1 + V_2 + \dots + V_k$ Wishart distribution [4mks]
- b) Let $X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ have a multinomial distribution where $X_1 = x_1$ and $X_2 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$
Let also $\mu = \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix}$ be the mean vector and $\Sigma = \begin{pmatrix} 7 & 3 & 2 \\ 3 & 4 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ be the variance-covariance matrix
compute [4mks]
- i. $E(X_1|X_2)$ [4mks]
 - ii. $Var(X_1|X_2)$
- c) Given that in part (a) above, if $X_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $X_2 = x_3$. Find [4mks]
- i. $E(X_1|X_2)$ [4mks]
 - ii. $Var(X_1|X_2)$

QUESTION 3: (20 Marks)

- a) Given that x has a trivariate normal distribution with mean $\mu' = (\mu_1 \ \mu_2 \ \mu_3)$ and $Var(x_i) = \sigma_i^2$ and $Cov(x_i, x_j) = \rho_{ij}\sigma_i\sigma_j \quad i \neq j = 1, 2, 3$

Show that the matrix of partial regression coefficient of x_1 given x_2 and x_3 is

$$B_{12.3} = \frac{\sigma_1}{\sigma_2} \begin{pmatrix} \rho_{12} - \rho_{13}\rho_{23} \\ 1 - \rho_{23}^2 \end{pmatrix} \quad [5mks]$$

- b) Suppose $X \sim N_p(\mu, \Sigma)$, $\Sigma > 0$ and y is given by $y = ax$

Find the characteristic function of y and show that $y \sim N_p(a'\mu, a'\Sigma a)$ [8mks]

- c) Suppose $X \sim N_p(\mu, \Sigma)$, $\Sigma > 0$. Show that $Q = (X - \mu)'\Sigma^{-1}(X - \mu)$ has a χ^2 -square distribution with p degrees of freedom [7mks]

QUESTION 4: (20 Marks)

- a) State and explain briefly the three levels of measurements in a multivariate statistics [6mks]

- b) Show that the mean of S is a bias estimator of Σ and hence give the unbiased estimate of Σ [4mks]

- c) Prove that $f(x) = \frac{e^{-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu)}}{|\Sigma|^{\frac{1}{2}}(2\pi)^{\frac{p}{2}}}$, where $-\infty < x_i < \infty$ and $-\infty < \mu_i < \infty$, $i = 1, 2, \dots, p$ is a probability density function [5mks]

- d) Suppose $X \sim N_p(\mu, \Sigma)$, $\Sigma > 0$, find the characteristic function ($Q_X(t)$) of X [5mks]

QUESTION 5: (20 Marks)

- a) Given the multivariate density function x with the distribution $x \sim N_p(\mu, \Sigma)$, obtain its moment generating function [5mks]

- b) Let the random variable $x \sim N_3(\mu, \Sigma)$ with $\mu' = (2 \ -3 \ 1)$ and $\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$.

Find the distribution of $3x_1 - 2x_2 + x_3$ [6mks]

- c) Show how a standardized P -dimensional normal variable y is obtained from a P -dimensional variable x with mean μ and covariance matrix Σ [4mks]

- d) Suppose $x \sim N_2(\mu, \Sigma)$ where $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$

Obtain the first and second principal components [5mks]