

15



(Knowledge for Development)

KIBABII UNIVERSITY
MAIN EXAMINATION

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS AND EDUCATION SCIENCE/ARTS (SB)

COURSE CODE: MAT 301

COURSE TITLE: REAL ANALYSIS II

DATE: 05/12/18

TIME: 8.00-10.00AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

Question One:(30 marks)

- (a) Define the following terms:
- (i) Interior point of a set [2 marks]
 - (ii) Point of contact of a set [2 marks]
 - (iii) Complete metric space [2 marks]
- (b) Let $(X_1, \rho_1), (X_2, \rho_2)$ be metric space and if $X = X_1 \times X_2$, define ρ on $X \times X$ by $\rho((X_1, X_2)(Y_1, Y_2)) = \rho_1(X_1, Y_1) + \rho_2(X_2, Y_2)$. Show that X, ρ is a metric space. [7 marks]
- (c) Show that a set E is open if and only if $E = E^0$, where E^0 is an interior point of E . [6 marks]
- (d) Show that the limit of a convergent sequence is unique in a metric space [7 marks]
- (e) Proof that the closure of the set X is equal to the union of the set X and the derived set of X [4 marks]

Question Two:(20 marks)

- (a) Show that every open neighbourhood is an open set. [7 marks]
- (b) Show that derived set is closed in a metric space. [7 marks]
- (c) Proof that a set E is closed in a metric space if and only if its complement is open in a metric space. [6 marks]

Question Three:(20 marks)

- (a) Show that the interior of a set E is open [6 marks]
- (b) State and proof Cauchy - Schwarz inequality [9 marks]
- (c) Let X, ρ be a metric space and (x_n) be a sequence of points which is Cauchy. Then (x_n) is bounded i.e the range (x_n) is a bounded subset [5 marks]

marks]

Question Four:(20 marks)

- (a) Show that if (X, ρ) is a metric space, then:
- (i) \emptyset, X are open in (X, ρ)
 - (ii) If $(E_\alpha : \alpha \in \Omega)$ is a family of open subsets of (X, ρ) then, $\bigcup_{\alpha \in \Omega} E_\alpha$ is open in (X, ρ)
 - (iii) If $\{E_1, E_2, \dots, E_n\}$ is a family of open subsets of (X, ρ) , then: $\bigcap_{i=1}^n E_i$ is open in (X, ρ) [12 marks]
- (b) Show that a subset A of a real number is closed if and only if whenever $a_n \in A$ for all $n \in \mathbf{N}$, and $a_n \rightarrow c$, we have $c \in A$. [8 marks]

Question Five: (20 marks)

- (a) Define the following terms with respect to a metric space:
- (i) Open set [2 marks]
 - (ii) Limit point [2 marks]
- (b) Show that a subset A of X is closed if and only if it contains all its limit points. [4 marks]
- (c) Let (X, ρ_1) and (Y, ρ_2) be metric spaces, then a function $f : X \rightarrow Y$ is continuous at the point $x \in X$ if and only if for any open neighbourhood V of $f(x)$ in Y , there is a neighbourhood U of x such that $f(U) \subseteq V$ [12 marks]

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