Mathematical Model for Predicting Climate Change Impact on Water Availability and Quality: A SWAT-SEIR Framework

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Abstract- Flood-prone regions, such as Budalangi in the Lake Victoria Basin, frequently experience endemic contamination of their water resources due to recurrent flooding events. Consequently, effective water management strategies in such regions must address not only acute contamination during flood events but also the persistent, chronic nature of waterborne contamination driven by climatic variability and infrastructural challenges. The framework successfully demonstrated the capacity to simulate the dynamic behavior of water systems influenced by environmental stressors, making it suitable for comprehensive climate-water interaction assessments. This paper presents an integrated mathematical model that captures the dynamics between climate change and water security and analyze the impact of climate change on water security in Budalangi.

Index Terms- Climate change, water security, contamination propagation, hydrological modeling, basic reproduction number, RK4

I. INTRODUCTION

This study develops an integrated mathematical model to examine the impact of climate change on water security in flood plain Budalangi. The model tracks water as it transitions through four key compartments each representing a distinct hydrological state and process drawing structural inspiration from compartmental epidemiological models but adapted for environmental analysis. The Susceptible (S) compartment represents the volume of clean, available water, including fresh precipitation and uncontaminated surface and near-surface flows. The Exposed (E) compartment comprises water that has recently contacted potential contaminants such as through climate-driven runoff events, surface interactions, or shallow infiltration placing it at risk of becoming polluted. The Infectious (I) compartment captures actively contaminated water, which may arise via conversion from the exposed state or through direct inputs of pollutants (e.g., from agricultural or urban runoff and waste influenced by climate extremes). The Recovered (R) compartment accounts for water that has undergone natural purification processes (percolation, biodegradation, and filtration) or has been treated through engineered interventions, rendering it again safe for use [32]. Central hydrological processes reflecting climate influence are incorporated explicitly as, Precipitation (P) which is the principal climate-driven input, directly replenishing the clean water pool, Contamination transfer (αSI) which is a nonlinear contact term representing the risk that clean water becomes exposed via interaction with infected (contaminated) water, a process that intensifies under conditions such as flooding or increased water mixing. Pollutant loading (PL) represents direct, often episodic, external inputs of contamination, such as those intensified during storms and floods or as a result of land use affected by climate variability. Losses ($\mu_S S$, $\mu_E E$, $\mu_I I$, $\mu_R R$) represent all natural outflows and removals for each compartment, encompassing evaporation, seepage, abstraction and downstream transport; many of these rates can fluctuate with climatic conditions. Remediation and recovery (γ, δ) represent both natural self-purification capacity (e.g., dilution, decay) and engineered measures (e.g., treatment plants) to restore contaminated water [50].

These dynamics are governed by first-order differential equations, which facilitate the analysis of how climate-induced stresses propagate through the water resource system.

II. MODEL ASSUMPTION

To ensure the model reflexively captures real climatic variability and hydrological behavior observed in Budalangi, the following scientifically grounded assumptions are adopted.

i. Climate-Driven Forcing
Precipitation and temperature are modeled as time-dependent functions based on observed or climatological records, thereby representing wet and dry seasonality

ii. First-Order Kinetics
Water and pollutant transfers between all model compartments occur at rates proportional to the current compartment size, as is standard in environmental fate modeling.

iii. Surface Flow Dominance
Surface runoff and rainfall are treated as the main drivers for water and pollutant movement; subsurface and groundwater flows are included within generalized loss terms to maintain parsimony.

iv. Compartment Homogeneity
Each compartment (Susceptible, Exposed, Infectious, Recovered) is assumed to be uniformly mixed at the catchment scale and time step of the model.

III. COMPARTMENTAL DIAGRAM

To better understand the interactions within the water contamination dynamics, we represent the enhanced SWAT-SEIR model using a compartmental diagram [8]. Each compartment represents a state of water quality: Susceptible (safe), Exposed (at risk), Infectious (contaminated) and Recovered (remediated or naturally cleansed). Arrows between compartments depict the influence by environmental and anthropogenic processes such as precipitation, contamination and treatment.

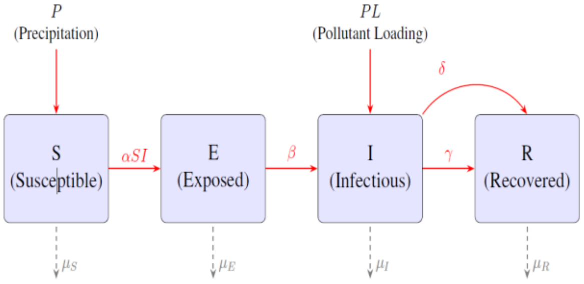

Figure 1. The SWAT-SEIR Compartmental Diagram

IV. MATHEMATICAL MODEL FORMULATION

We consider the following system of nonlinear differential equations:

$$\begin{aligned}
\frac{dS}{dt} &= P(t) - \alpha(t)SI - \mu_S(t)S \\
\frac{dE}{dt} &= \alpha(t)SI - \beta(t)E - \mu_E(t)E \\
\frac{dI}{dt} &= \beta(t)E + PL(t) - \gamma(t)I - \delta I - \mu_I(t)I \\
\frac{dR}{dt} &= \gamma(t)I + \delta I - \mu_R(t)R
\end{aligned} \tag{1}$$

This framework allows dynamic adjustment of model behavior to climate trends and supports projections of water quality and resource dynamics under both present and future climate scenarios.

Table 1: Model Variables and Parameters

Symbol	Description
$S(t)$	Volume of clean, uncontaminated water (Susceptible)
$E(t)$	Volume of water exposed to pollution but not yet infectious (Exposed)
$I(t)$	Volume of contaminated/infectious water (Infectious)
$R(t)$	Volume of remediated or recovered water (Recovered)
P	Rate of precipitation input (constant average)
αSI	Rate at which contamination is transmitted
β	Rate at which exposed water becomes infectious
$P L$	External pollutant loading (e.g., runoff, waste discharge)
γ	Natural recovery rate of contaminated water
δ	Engineered remediation rate (e.g., treatment systems
μ_S , μ_E , μ_I , μ_R	Natural loss rates due to evaporation, seepage, and infiltration from compartments

All parameters are nonnegative and mostly treated as constants for the analysis here. All parameters may be time-dependent and are influenced by rainfall, temperature, land use and seasonality.

V. INTERPRETATION AND APPLICATION

This compartmental model provides a dynamic framework for tracking the fate of water and associated pollutants within a catchment or water resource system, explicitly considering both natural and engineered processes such as precipitation inputs, surface runoff, contamination events, remediation and various loss pathways [8]. Each model parameter including rates of exposure, contamination, recovery and losses can be empirically measured, estimated from monitoring data, or calibrated using observed hydrological and water quality records. This flexibility enables the model to be tailored to the specific characteristics and data constraints of different watersheds. Furthermore, the structure of the model allows for extension or simplification depending on research objectives or data availability. For instance, the natural recovery parameter (γ) maybe omitted if bioremediation is negligible, or the exposure rate (α) can be modeled as a time-dependent function to realistically capture seasonal variability in runoff. This SEIR-inspired approach supports scenario analysis, risk assessment and management planning by capturing the coupled effects of climate, hydrology, Pollution and intervention strategies on water security. This climate-driven compartmental model enables scenario and sensitivity analysis to forecast both short-term pollution events and long-term water quality trends under projected climate change.

VI. MAPPING OF CLIMATE DATA TO MODEL PARAMETERS

This mapping ensures that each parameter in the SWAT-SEIR framework dynamically responds to relevant climatic drivers. With parameters as explicit functions of time and climate, the resulting ODE system is:

$$\begin{aligned}
\frac{dS}{dt} &= P(t) - \alpha(t)SI - \mu_S(t)S \\
\frac{dE}{dt} &= \alpha(t)SI - \beta(t)E - \mu_E(t)E \\
\frac{dI}{dt} &= \beta(t)E + PL(t) - \gamma(t)I - \delta I - \mu_I(t)I \\
\frac{dR}{dt} &= \gamma(t)I + \delta I - \mu_R(t)R
\end{aligned} \tag{2}$$

This framework allows dynamic adjustment of model behavior to climate trends and supports projections of water quality and resource dynamics under both present and future climate scenarios [26].

VII. CLIMATE DRIVEN PARAMETARIZATION OF THE WATER MODEL

The compartmental model described captures the contamination dynamics in a water body influenced by climate and local environmental processes. Clean water is replenished primarily by rainfall but decreases due to contamination transmission from infectious water and natural losses. Contaminated water is modeled in stages: exposed water (recently contaminated but not infectious), infectious water (actively contaminated) and recovered water (remediated or naturally recovered). The model parameters such as contamination rates, recovery and losses allow the system to track pollutant fate considering natural and engineered processes. These parameters can be tailored based on empirical data to fit specific watershed characteristics and allow for flexible model complexity depending on the research or management needs.

Furthermore, climatic factors play a key role by influencing parameters such as rainfall, contamination transmission and pollutant loading. Rainfall is modeled as a time-dependent function affecting contamination rates and pollutant inflows, with high rainfall periods corresponding to increased contamination and pollutant transport. Climate variables like temperature and evapotranspiration also impact natural remediation and water losses. By integrating climate data into the model parameters dynamically, the framework supports realistic simulations of seasonal, inter-annual and long-term water quality trends, facilitating scenario analysis and risk assessment under climate change.

ACKNOWLEDGMENT

Special thanks to Dr Boniface Otieno Kwach for his intellectual guidance and for sharing his expertise in mathematical modeling. We also thank the Kenya Meteorological Department for the data used in the study.

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