



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
FIRST-YEAR SECOND SEMESTER
SUPPLEMENTARY/SPECIAL EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE
MATHEMATICS

COURSE CODE: MAA 122/MAT 122

TITLE: ELEMENTARY APPLIED MATHEMATICS

DATE: 17/8/2023

TIME: 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question **One** and **Any other TWO** Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages Please Turn Over

QUESTION 1 (30 MARKS)

- a) Find the angle between planes $3x + 2y - 2z = 4$ and $2x + y + 2z = 1$ (3marks)
- b) Find the centre and radius of the circle passing through the points $(4, 3)$, $(0, 1)$ and $(1, 0)$. (6 marks)
- c) Determine if the line with parametric equations $x = -2t$, $y = 2 + 7t$, $z = -1 - 4t$ intersects the plane $4x + 9y - 2z = -8$. If it does give the coordinates of that point of intersection. (4 marks)
- d) Find Cartesian equation for the curve $r = 2 \sin \theta + 2 \cos \theta$ (3marks)
- e) For what values of x are the vectors $\langle -6, x, 2 \rangle$ and $\langle x, x^2, x \rangle$ orthogonal? (4 marks)
- f) Given $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$, $\mathbf{b} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{c} = 3\mathbf{i} - \mathbf{k}$, find the angle between $\mathbf{a} + \mathbf{b}$ and $\mathbf{b} - 2\mathbf{c}$. (6 marks)
- g) A ball of mass 72 g , travelling horizontally at 39 ms^{-1} strikes a wall at right angles and rebounds with a speed of 28 ms^{-1} . Find the impulse exerted on the ball. (4marks)

QUESTION 2 (20 MARKS)

- a) Derive the equation of motion $S = ut + \frac{1}{2}at^2$. (4 marks)
- b) A stone is projected vertically upwards with a velocity of 90 m/s from the ground. Calculate:
- The time it takes to reach the maximum height. (2 marks)
 - The time of flight. (3 marks)
 - The maximum height reached. (3 marks)
 - The velocity with which it lands on the ground. (3 marks)
- c) Show from Newton's second law of motion that Force = mass $\times$ acceleration. (5 marks)

QUESTION 3 (20 MARKS)

- a) Write down the equation of the plane containing the points $P(4, -3, 1)$, $Q(-3, -1, 1)$ and $R(4, -2, 8)$. (5 marks)
- b) Find the distance between the parallel planes $10x + 2y - 2z = 10$ and $5x + y - z = 4$ (5 marks)
- c) i) Show that $\mathbf{a} \times \mathbf{a} = \mathbf{0}$ for any vector $\mathbf{a}$. (4 marks)
- ii) Prove that the vector $\mathbf{a} \times \mathbf{b}$ is orthogonal to vector $\mathbf{b}$. (6 marks)

QUESTION 4 (20 MARKS)

- a) Is the point $(1, -1)$ within or outside the circle whose equation is $4x^2 + 4y^2 + 12x - 20y + 25 = 0$ (5 marks)
- b) Show that the equation of a circle that passes through $(-1, 1)$ and whose centre is the point of intersection of the lines $x + 3y + 7 = 0$ and $2y - 3x + 12 = 0$ is given by $x^2 + y^2 - 4x + 6y - 12 = 0$ (6 marks)
- (c) (i) Show that a particle has the same velocity at a height of S metres above the ground when it is ascending and when it is descending. (5 mks)
- (ii) Show that if a body leaves with a velocity of 10 m/s from the ground, it will arrive at the same spot with the same velocity downwards. (4 mks)

QUESTION 5 (20 MARKS)

- a) Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates (4 marks)
- b) Convert $r = -7 \cos \theta$ into Cartesian equation. (4 marks)
- c) Find the area enclosed by one loop of the four-leaved rose $r = \sin 2\theta$ (6 marks)
- d) Calculate the arc length of the polar curve $r = -4 \sin \theta$, $0 \leq \theta \leq \pi$ (6 marks)