200

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2024/2025 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAA 321/MAT 304**

COURSE TITLE: **COMPLEX ANALYSIS 1**

DATE: 22/04/2025 **TIME: 11:00 AM – 1:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 2 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Determine the two square roots of $z = 5 + 12i$ in polar form (4mks)

b) Solve the equation $x^2 + 1 = 0$ (3mks)

c) If $z = 1 - i$ determine z^{10} in polar and Cartesian forms (5mks)

d) Express the function $f(z) = 5z^3 + 2z^2 - z + i$ in the form $f(z) = u + iv$ and verify the Cauchy Riemann equations for the function. (5mks)

e) Evaluate $\int (x^2 + ixy)\ dz$ from $A(1,1)$to $B(2,4)$along the curve $x = t, y = t^2$ (6mks)

f) Find the first four terms of the Taylor series expansion of $f(z) = \frac{z+1}{(z-3)(z-4)}$ about the point $z = 2$. Find the region of convergence. (5mks)

g) Find the value of w corresponding to $z = 2 - i$ if $w = f(z) = z(3 - z)$ (2mks)

QUESTION TWO (20MARKS)

a) Show that $coth^{-1} z = \frac{1}{2}\ln\left(\frac{z+1}{z-1}\right)$ (5mks)

b) Given $v(x,y) = e^x(x\cos y - y\sin y)$ as the imaginary part of analytic function, $f(z) = u + iv$ and hence find $u(x,y)$ (8mks)

c) Prove that $\cos(z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2$ (5mks)

d) Investigate whether $f(z) = 2z^2 - 3z$ is continuous at a point $z_0 = -1$ (2mks)

QUESTION THREE (20MARKS)

a) Evaluate $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$ along

   i. The parabola $x = 2t, y = t^2 + 3$

   ii. Straight lines from $(0,3)$ *to* $(2,3)$ and then from $(2,3)$ *to* $(2,4)$

   iii. Straight lines from $(0,3)$ *to* $(2,4)$ (14mks)

b) Use the Cauchy integral formula to evaluate $\int_c \frac{z+4}{z^2+2z+5} dz$ where c is the circle $|z + 1 - i| = 2$ (6mks)

QUESTION FOUR (20MARKS)

a) Find the Laurent series expansion for (10mks)

$f(z) = \frac{1}{z(z-1)}$

   i. $0 < |z| < 1$

   ii. $|z| > 1$

b) Find the residues of the function $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ hence evaluate $\int_c f(z)dz$ (10mks

QUESTION FIVE (20MARKS)

a) Determine the complex number z if $\arg(z + 1) = \frac{\pi}{6}$ and $\arg(z - 1) = \frac{2\pi}{3}$ (10mks)

b) Examine the nature of the function $f(z) = \frac{x^2y^5(x+iy)}{x^4+y^{10}}$; $z \neq 0, f(0) = 0$(10mks)