(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST-YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR SCIENCE

COURSE CODE: **MAP 122/MAT212**

COURSE TITLE: LINEAR ALGEBRA I

DATE: 17/8/2023 **TIME:** 11 AM -1:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over

QUESTION ONE (30marks)

a). Define the following terms

(i). Diagonal matrix (1 mk)

(ii). Parallel vectors (1 mk)

(iii). A vector (1 mk)

(iv). A spanning set of vectors (2 mks)

b). Compute the inverse of the matrix $M = \begin{bmatrix} 4 & 6 & -5 \\ 5 & 7 & 0 \\ 2 & 1 & 4 \end{bmatrix}$ (8 mks)

c). Find the vector equation of a plane passing through points $A(1,3), B(0,-1)$ and $C(2,1)$. (5 mks)

d). Solve the system $x + y - 2z = 1;\ y - z = 3;\ -x + 4y - 3z = 14$ (7 mks)

e). Prove that the image of a transformation $T: V \to W$ is a subspace of W. (5 mks)

QUESTION TWO (20marks)

a). A straight line passes through $A(2,1,0)$ and $B(3,5,1)$. Find the parametric equation and the symmetric form of the line. (6 mks)

b).(i). What is a nonsingular matrix? If 4×4 matrix is nonsingular, what could its rank be? (3 mks)

(ii). Show that the following linear system has an infinite number of solutions,

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$ (6 mks)

c). Determine whether the set V= {[1 0 0], [0 1 0], [−1 − 1 0]} is a linearly independent set of vectors. (5 mks)

QUESTION THREE (20marks)

a). Find a unit vector in the direction of vector $\boldsymbol{w} = (-10, 5, -10)$. (3 mks)

b).(i) What is a linear combination of vectors in $\mathbb{R}^n$? (2 mks)

(ii). Express $\boldsymbol{b} = (3, 6, 9)$ as a linear combination of $\boldsymbol{v_1} = (1,2,3)$, $\boldsymbol{v_2} = (3,5,1)$ and $\boldsymbol{v_3} = (0,0,8)$. (6 mks)

c). Find the projection of vector $v = (4, -1, -1)$ in the direction of $u = (2,0, -5)$. (4 mks)

d). Consider the transformation T given by $T(x_1, x_2) = (7x_1 + 4x_2, -2x_1 - 3x_2)$. Find the inverse transformation T^{-1}. (5 mks)

QUESTION FOUR (20marks)

a). Suppose we know for a linear transformation T of $\mathbb{R}^2$ that $T\begin{bmatrix}-1\\6\end{bmatrix} = \begin{bmatrix}8\\4\end{bmatrix}$ and $T\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}8\\-12\end{bmatrix}$. Find the matrix $\boldsymbol{A}$ so that $T(x) = \boldsymbol{A}x$. (7 mks)

b). Consider the matrices $\boldsymbol{u_1} = (1,2,0), \boldsymbol{u_2} = (0,4,2)$ and $\boldsymbol{u_3} = (-1,3,2)$ in $\mathbb{R}^3$. Prove that the set $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\}$ spans $\mathbb{R}^3$. (8 mks)

c). Prove that the kernel of a transformation $A: X \to Y$ is a subspace of X. (5 mks)

QUESTION FIVE (20marks)

a). Find the vector and parametric equations of a plane passing through point $P(-2,1,2)$ and normal to vector $n = (3,2,-7)$. (5 mks)

b). Let $A = \begin{bmatrix}1 & 4 & -3\\6 & 3 & 0\end{bmatrix}$, $B = \begin{bmatrix}3 & 2 & 1\\-2 & -6 & 5\end{bmatrix}$, $C = \begin{bmatrix}2 & 4\\4 & 0\\-2 & 2\end{bmatrix}$ and $D = \begin{bmatrix}1 & -1\\2 & 4\\-1 & 3\end{bmatrix}$. Let $\lambda = 2\beta$ and $\beta = 2$. Compute the following matrices

(i). $T = \lambda^2 A + \beta B - C^T$ (5 mks)

(ii). The inverse of DB. (4 mks)

c). Prove that the vectors $\boldsymbol{u} = (2,1,1), \boldsymbol{v} = (-1,-1,1)$ and $\boldsymbol{w} = (1,2,3)$ are linearly independent. (6 mks)