



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2024/2025 ACADEMIC YEAR
FIRST YEAR FIRST SEMESTER
MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN
APPLIED MATHEMATICS

COURSE CODE: MAT 851

COURSE TITLE: ODE I

DATE: 22/04/25

TIME: 8 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE [20 MARKS]

- (a) Describe the following: (2 mks)
- (i) Orbit (2 mks)
- (ii) Continuation of solution (5 mks)
- (b) Discuss the stability of the critical point at the origin of the system
- $$\begin{cases} x' = x + y + x^2 \\ y' = -x \end{cases}$$
- (c) By computing appropriate Lipschitz constant show that the following function satisfy Lipschitz conditions on the set S (5mks)
- $$f(x, y) = 25x^2 + y^2 \text{ on } S = \{(x, y) / |x| \leq 1, |y| \leq 1\}$$
- (d) Let $I \subseteq \mathbb{R}$ be an interval with $\tau \in I$ and let $U: \rightarrow [0, \infty)$ be differentiable. If there is a non-negative constant K such that $|U'(t)| \leq KU(t)$, for all $t \in I$ prove that (6 mks)
- $$U(\tau)e^{-K(t-\tau)} \leq U(t) \leq U(\tau)e^{K(t-\tau)}$$

QUESTION TWO [20 MARKS]

Consider the IVP $y' = 2y - 3x - 1, y(0) = -1$

- (a) Show that all the successive approximations $\Phi_0, \Phi_1, \Phi_3 \dots$ exists for all real x (5 mks)
- (b) Compute the first four approximations (11mks)
- (c) Compute the exact solution (4mks)

QUESTION THREE [20 MARKS]

- (a) State conditions that satisfy a Lyapunov function (3mks)
- (b) Consider $\begin{cases} x' = -x + y \\ y' = -x - y^3 \end{cases}$
- Show that $V(x, y) = 2x^2 + 2y^2$ is a strict Lyapunov function for (0,0) (5 mks)
- (c) Given the equation $\begin{cases} x' = y - xy^2 \\ y' = -x^3 \end{cases}$ (6 mks)
- Discuss the stability of the critical point at the origin
- (d) Consider $\begin{cases} x' = 3x + y^3 \\ y' = -2y + x^2 \end{cases}$ and the function $V(x, y) = x^2 - y^2$ show that the origin is unstable (6mks)

QUESTION FOUR [20 MARKS]

Given the initial value problems find in each case an interval on which a solution exists

(a) $\begin{cases} y' = y^2 \\ y(-1) = 1 \end{cases}$ (8 mks)

(b) $\begin{cases} y'(x) = \frac{e^{y^2}-1}{1-x^2y^2} \\ y(-1) = 2 \end{cases}$ (12mks)

QUESTION FIVE [20 MARKS]

(a) Describe (4 mks)

(i) A regular path

(ii) Index

(b) State the Grobmann-Hartman theorem (4mks)

(c) Consider the equation

$$(x, y)' = (y, -x)$$

(i) Rewrite in polar coordinates (5 mks)

(ii) Sketch the phase portrait. (2 mks)

(d) Find the exponential of a matrix $A = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$ where I is identity (5 mks)