RENDICONTI DEL CIRCOLO MATEMATICO DI PALERMO
Serie II, Tomo XLIX (2000), pp. 141-150

ON CONVERGENCE OF SECTIONS OF SEQUENCES IN BANACH SPACES

SHEM AYWA - JAN H. FOURIE

An elementary proof of the (known) fact that each element of the Banach space $\ell^p_w(X)$ of weakly absolutely p-summable sequences (if $1 \leq p < \infty$) in the Banach space X is the norm limit of its sections if and only if each element of $\ell^p_w(X)$ is a norm null sequence in X, is given. Little modification to this proof leads to a similar result for a family of Orlicz sequence spaces. Some applications to spaces of compact operators on Banach sequence spaces are considered.

1. Introduction and notation.

The scalar sequence space Λ will throughout assumed to be a normal BK-space with the AK-property. This means that Λ has the following properties:

(a) It is a Banach space such that the coordinate projections are continuous.

(b) If $(\lambda_i) \in \Lambda$ and $|\alpha_i| \leq |\lambda_i|$ for $i = 1, 2, \ldots,$ then $\|(\alpha_i)\| \leq \|(\lambda_i)\|$.

(c) Each $(\lambda_i) \in \Lambda$ is the norm limit of its sections. Thus $(\lambda_i) = \lim_{n\to\infty} (\lambda_i)(\leq n)$ where $(\lambda_i)(\leq n) = (\lambda_1, \lambda_2, \ldots \lambda_n, 0, 0, \ldots)$, or in other words $\lim_{n\to\infty} (\lambda_i)(\geq n) = 0$ where $(\lambda_i)(\geq n) = (0, 0, \ldots 0, \lambda_n, \lambda_{n+1}, \ldots)$.

Key words and phrases. weak ℓ^p-sequences; Orlicz sequence spaces; compact operators. Financial assistance from the FRD and PU for CHE is greatly acknowledged.

1991 *Mathematics Subject Classification.* 46B45, 46A32, 47B37.

Equivalently, the set of unit vectors $\{e_n : n \in \mathbb{N}\}$ (where $e_n = (\delta_{i,n})_i$) is a Schauder basis for Λ. We may assume that $\|e_n\| = 1$ for all n. The sequence $(0, 0, \ldots, 0, \lambda_{n+1}, \lambda_{n+2}, \ldots \lambda_{m-1}, 0, 0 \ldots)$ is denoted by $((\lambda_i)(n < i < m))$.

When Λ has the above mentioned properties, then it is well known that the continuous dual space Λ^* is isomorphic to the Köthe dual space

$$\Lambda^\times := \left\{ (\alpha_i) \in \omega : \sum_{i=1}^{\infty} |\alpha_i \lambda_i| < \infty, \ \forall (\lambda_i) \in \Lambda \right\}$$

with respect to the obvious duality.

Let X be a Banach space with continuous dual space X^*. The closed unit ball in X will be denoted by B_X and sequences in X are denoted by (x_i), (y_i) etc. As in the case of scalar sequences we let

$$(x_i)(\leq n) := (x_1, x_2, \ldots x_n, 0, 0, \ldots)$$

and

$$(x_i)(\geq n) := (0, 0, \ldots, 0, x_n, x_{n+1}, \ldots).$$

The vector sequence space

$$\Lambda_w(X) := \{(x_i) \subset X : (a(x_i)) \in \Lambda, \forall a \in X^*\}$$

is a complete normed space with respect to the norm

$$\epsilon_\Lambda((x_i)) := \sup_{\|a\| \leq 1} \|(a(x_i))\|_\Lambda.$$

We put $\epsilon_p = \epsilon_\Lambda$ when $\Lambda = \ell^p$, the Banach space of p-absolutely summable scalar sequences (with $1 \leq p < \infty$). The closed linear subspace consisting of all sequences $(x_i) \in \Lambda_w(X)$ such that $(x_i) = \epsilon_\Lambda \lim_n (x_i)(\leq n)$ is denoted by $\Lambda_c(X)$. Hence

$$\Lambda_c(X) := \{(x_i) \in \Lambda_w(X) : \epsilon_\Lambda((x_i)(\geq n)) \to 0 \text{ if } n \to \infty\}.$$

It is well known (cf. [5] for a proof) that $(x_i) \in \Lambda_w^\times(X)$ if and only if $\sum_{i=1}^{\infty} \lambda_i x_i$ converges in X for every $(\lambda_i) \in \Lambda$, in which case the norm is