



KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2024/2025 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

COURSE CODE: SPH 813

COURSE TITLE: ELECTRODYNAMICS

DURATION: 3 HOURS

DATE: 23/01/2025

TIME: 2-4PM

INSTRUCTIONS TO CANDIDATES

- Answer ANY THREE QUESTIONS.
- Each question carries 20 MARKS.

• $\mu_0 = 4\pi \times 10^{-7} \text{ T.m/A}$, $\mu_0 = \frac{4\pi}{c}$

$$\vec{\nabla} \times \vec{B} \equiv \left(\frac{1}{r} \frac{\partial B_z}{\partial \varphi} - \frac{\partial B_\varphi}{\partial z} \right) \hat{r} + \left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} \right) \hat{\varphi} + \frac{1}{r} \left(\frac{\partial(rB_\varphi)}{\partial r} - \frac{\partial B_r}{\partial \varphi} \right) \hat{z}.$$

$$\vec{\nabla} \circ \vec{A} = \frac{1}{r} \frac{\partial}{\partial r} (rA_r) + \frac{1}{r} \frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

QUESTION ONE (20 MARKS)

- Use Biot-Savart law to show that the divergence of all magnetic fields is zero. (4marks)
- The magnetic field in a Cartesian region is given by $\vec{B} = B_0 \cos(kz - \omega t)\hat{j}$
 - Find the curl of the induced electric field at that location. (3marks)
 - If the E_z is known to be zero, find E_x . (4marks)
- Electric charge is uniformly distributed with a linear density λ along a semicircle of radius r . Determine the Coulomb force on a point charge Q placed at the center of curvature of the semicircle. (4marks)
- Show that the total field of a system of point charges is given by $E = \sum_i \frac{e_i}{R_i^3} \vec{R}_i$. (5marks)

QUESTION TWO (20 MARKS)

- Show that the electric field of an electrically neutral system is given by $E = \frac{3r(r \cdot D) - r^2 D}{r^5}$

Where symbols have their usual meaning. (5marks)
- Find the charge density at $x = 2m$ and $x = 5m$ if the electric field in the region is given by $\vec{E} = \begin{cases} ax^2 \hat{i} \frac{V}{M}, & 0 \leq x \leq 3m \\ bi \frac{V}{M}, & x > 3m \end{cases}$. (5marks)
- Electric charge is uniformly distributed with a density ρ inside a sphere of radius r . Determine the electric potential inside and outside the sphere. (5marks)
- Show that a change in the magnetic field is accompanied by the appearance of a rotational electric field and a change in the electric field. (5marks)

QUESTION THREE (20 MARKS)

- Electric charge is uniformly distributed with a linear density λ on an infinitely long line parallel to the y-axis. Determine the electric field strength at a point P a distance b from the center of the segment to the direction of the x-axis. (5marks)
- The expression for the magnetic field of a circular parallel plate capacitor is $\vec{B} = \frac{\mu_0 \Delta r}{2\pi R} e^{-\frac{t}{RC}} \left(\frac{r}{r_0}\right) \hat{\phi}$. Use this result to find the displacement current density between the plates. (5marks)
- Use gauge transformation to show that a vector potential A and a gradient of a scalar potential can be expressed in terms of current sources J and ρ where symbols have their usual meaning. (6marks)
- Find the rate of change with time of the magnetic field at a location at which the induced electric field is given by $\vec{E}(x, y, z) = E_0 \left[\left(\frac{z}{z_0}\right)^2 \hat{i} + \left(\frac{x}{x_0}\right)^2 \hat{j} + \left(\frac{y}{y_0}\right)^2 \hat{k} \right]$ (4marks)

QUESTION FOUR (20MARKS)

- a) Show that the e.m wave in a dielectric medium in x-direction can be expressed as one-dimensional wave equation $\frac{\partial^2 E_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}$. (5marks)
- b) Use the Poynting theorem to show that the field energy flux flowing through 1cm^2 in the direction perpendicular to the field vectors $\mathbf{E}$ and $\mathbf{B}$ is given by $\mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B})$ (6marks)
- c) A current $\mathbf{I}$ is uniformly distributed over a wire of a circular cross section with radius a . Find the volume current density $\mathbf{J}$. (2marks)
- d) A current $\mathbf{I}$ is uniformly distributed over a wire of a circular cross section with radius a
- Find the volume current density $\mathbf{J}$. (3marks)
 - Suppose the current density in the wire is proportional to the distance from the axis for some constant k , find the total current in the wire. (4marks)

QUESTION FIVE (20 MARKS)

- a) Show that the magnetic field at the center of a solenoid of n turns is given by $\mathbf{B} = \mu_0 n \mathbf{I}$ (3marks)
- b) Use the energy conservation law of electromagnetic fields to show that the flow of energy through a closed surface S surrounding a volume V has an energy flux characterized by a vector $\boldsymbol{\sigma} = \frac{c}{4\pi} (\mathbf{E} \times \mathbf{H})$. (5marks)
- c) A uniform electric field $\mathbf{E}$ is at right angles to the magnetic field. Suppose that $\mathbf{B}$ points to x-direction and $\mathbf{E}$ z-direction. Determine the path taken by a positive charge released from the origin given $y(t) = C_1 \cos \omega t + C_2 \sin \omega t + \left(\frac{E}{B}\right)t + C_3$
 $z(t) = C_2 \cos \omega t - C_1 \sin \omega t + C_4$ (7 marks)
- d) Derive the Gauss divergence law by considering a point charge q enclosed by a surface S (5marks)