(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE: MAP 314

COURSE TITLE: NUMBER THEORY

DATE: 05/12/2023 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following

i. Greatest Common Divisors (2marks)

ii. The space $\mathbb{Q}[x]$ (2marks)

iii. Primitive polynomial (2marks)

iv. Diophantine Equation (2marks)

b) Prove that if $a \equiv b \ mod \ n$ and $c \equiv d \ mod \ n$, then $ac \equiv bd \ mod \ n$ (5marks)

c) The neighborhood theater charges Ksh180 for adult admissions and Ksh75 for children. On a particular evening the total receipts were Ksh9000. What is the maximum number of people who may have attended? (15marks)

d) State the Division Algorithm (2marks)

QUESTION TWO (20 MARKS)

a) State four addition axioms satisfied by the ring of integers, $\mathbb{Z}$ (4marks)

b) Prove that $\sqrt[3]{2}$ is not a rational number (7marks)

c) Solve the Congruence $8x \equiv 13 \ mod \ 29$ (7marks)

d) Prove that if $a \equiv b \ mod \ n$ then $b \equiv a \ mod \ n$, then $a, b, n \in \mathbb{Z}$ (2marks)

QUESTION THREE (20 MARKS)

a) State the Eisenstein Irreducibility Criterion (EIC) (2marks)

b) Discuss if the polynomial $f(x) = 2x^4 + \frac{3}{7}x^3 - 2x^2 - x + 3$ is irreducible or not (3marks)

c) Prove that if $n \in \mathbb{Z}$, then n^2 does not leave a remainder of 2 or 3 when its divided by 5 (8marks)

d) Prove that an integer p is prime iff it is irreducible (4marks)

e) If $a, b, c \in \mathbb{Z}$, $a \neq 0$ and $ab = ac$, prove that $b = c$ (3marks)

QUESTION FOUR (20 MARKS)

a) Let p be a prime and suppose $p \nmid a$ the prove that $a^{p-1} \equiv 1 \ mod \ p$ (8marks)

b) Prove that $\frac{1}{1.2} + \frac{1}{2.3} + \cdots + \frac{1}{n.(n+1)} = \frac{n}{n+1}$ for every positive integer n (8marks)

c) If $m, n \in \mathbb{Z}$, if $m > 0$ and $n < 0$, prove that $mn < 0$ (4marks)

QUESTION FIVE (20 MARKS)

a) Solve the Diophantine equation $11x + 13y = 369$ (8marks)

b) If $n \in \mathbb{Z}$, prove that $0.n = 0$ (5marks)

c) Simplify $146!\ (mod\ 149)$ to a number in the range $\{0,1,\ldots,148\}$ (7marks)