Gauge-Invariant Wigner Function Extended to High Temperature Superconductivity

Abel Mukubwa
Department of Science Technology and Engineering
Kibabii University, P.O Box 1699 – 50200, Bungoma, Kenya.
Tel (Mobile phone): (+254) 717 285 762
e-mail: abelmuwa@gmail.com

John Wanjala Makokha
Department of Science Technology and Engineering
Kibabii University, P.O Box 1699 – 50200, Bungoma, Kenya.

Abstract

The realization of gauge-invariant Wigner function (GIWF) has revolutionized studies on quantized and classical electromagnetic fields and has been adapted to a magnetostatic phenomenon in superconducting systems. We apply the quantum fluid moment hierarchy equations in solving the conservative moment equation of the gauge-invariant Wigner operator. The results show that the lower critical field (H_{c1}), the vortex radius (r_0) and the penetration depth (λ) show dependence on Cooper pair excitation energy (E_k).

Keywords: *Wigner operator, Wigner function, flow velocity, vortex radius, penetration depth, lower critical field*

1. Introduction

It remains undisputed that a Cooper pair plays a central role in high temperature superconductivity [1]. Previously, research has divulged the coexistence between zero- and finite-momentum Cooper pairs [2]. The existence of finite-momentum Cooper pairs (electronic composite bosons) alludes to a possible boson-electron interaction. A system that embodies an electronic boson interacting with a fermion in Bose-Einstein condensate (BEC) state was first proposed by Tomalchev [3]. A pseudogap occurs simultaneously as pre-formed finite-momentum Cooper pairs, which were neglected in entirety by the conventional BCS theory [4]. Interactions between a Cooper pair (boson) and a fermion (electron) has been studied in high temperature superconductors in the pseudogap [4] and in superconducting state [5]. Ufrecht et al. [6] argue that under some conditions, attraction between a boson and a fermion is possible. The strength of pairing in condensate state is determined by unusual electronic excitations – plasmons [7]. During B–F interaction, the boson-fermion separation shrinks to weaken the pair breaking ability of the Coulomb effect, which vanishes at T_c [8]. The phenomenon of high temperature superconductivity is driven by the collective behaviour of boson-fermion pairs rather than a single-particle-like behaviour of the condensate [9]. It is our view, in this study, that a boson-fermion-pair condensate (being a composite fermion condensate) is shielded from the magnetic flux by bosonic nonsuperconducting particles – Cooper pairs. It has been shown that the collective excitations of boson-fermion pair condensates (BFPC) are in the order of the energy gap [5].

Excitation of charges with energy much below that of the Fermi electron – acoustic plasmons - have observed in both electron- and hole-doped cuprates using resonant inelastic X-ray scattering (RIXS) [10][11[12]. The observed plasmons are predominantly associated with O sites of the copper oxide plane [13]. Plasmons in HT superconductors originate from singularity of long-range Coulomb interactions in the limit of long wavelength [14] and contribute significantly to the pairing mechanism [15][16]. Surface-plasmon dispersion at the interface of two media are isotropic, bulk-plasmon dispersions are anisotropic. Various quantum theories have been adopted in the study of high temperature superconductivity [HTS]. For instance, gauge-dependent functions have previously been adapted to the phenomenon of superconducting systems in the presence of some generic Abelian (like electromagnetic) and non-Abelian (like spin and particle-hole) gauge-fields to study the transport and magneto-electric fields [17][18]. However, their gauge-dependence pose a challenge in solving equations that involve the functions. In an effort to obtain a quantitative valuation of the physical quantities in a superconducting system, we adopt the gauge-invariant Wigner operator (GIWO) and the gauge-invariant Wigner function (GIWF) of a non-relativistic charged particle in the presence of fully classical fields.

The Wigner function (WF) and quasiprobability distributions allow quantum-mechanical expectation values to be written as phase-space integrals analogous to those of classical mechanics (CM) [19]. In such instances, classical intuitions may then be taken over to quantum mechanics (QM). The WF may or may not be dependent on the gauge potentials. A gauge-invariant Wigner operator (GIWO) and a gauge-invariant Wigner function (GIWF) that allow for both quantized and classical electromagnetic fields were introduced by Serimaa *et al.* [19] and within the postulate that physical observables can be measured without referring to the gauge potentials. In fact, GIWF is the quantum equivalent of classical particle distribution function and can be used to calculate the average values of physical observables [20]. GIWF, as a quantum tool, enhances the quantum kinetic theory in studying quantum plasmas, condensed matter systems, and, in general, perturbed quantum systems.

In developing the (GIWF) from a GIWO, quantum-mechanical expectation values of operators are computed, just like a classical expected value, as an integral over phase-space under some conditions [19]. In that respect, all classical phase-space distributions are not valid Wigner functions. For example, $W(r,p) = \delta(r)\delta(p)$ is feasible in CM but would give $\Delta p \Delta x = 0$ in QM.

The GIWF for quantum distribution is defined as [19]

$$W(\mathbf{r},\mathbf{v},t) = \left\{ \begin{array}{c} \left(\frac{1}{2\pi\hbar}\right)^3 \int d^3\mathbf{s} \exp\left[\frac{i}{\hbar}\mathbf{s}\cdot\left(m\mathbf{v} + q_s\int_{-1/2}^{1/2} d\tau\mathbf{A}(\mathbf{r}+\boldsymbol{\tau}\mathbf{s},t)\right)\right] \\ \times\,\psi^*\left(\mathbf{r}+\frac{1}{2}\mathbf{s},t\right)\psi\left(\mathbf{r}-\frac{1}{2}\mathbf{s},t\right) \end{array} \right\} \quad (1)$$

Where $\mathbf{r}$, $\tau = (t_1 - t_2)$ and $t = (t_1 + t_2)/2$ are the position, kinetic momentum, time lag and the average time. The wave function is assumed to be normalized to unity. In addition, $\hbar = h/2\pi$ is the reduced Planck's constant, $\mathbf{A}$ ($\mathbf{r}$, t) is the vector potential while m and q are the mass and charge of a Cooper pair in a state described by a wave function $\psi(\mathbf{r},t)$. The extra integral in Eq. (1) containing the vector potential compensates for the change in the wave function in a local gauge transformation. The use of a non-covariant, one-time pseudo distribution renders the interpretation issues of W less obscure than in a four-dimensional space-time version [21]. The equation of motion of a charged particle in a magnetic field in form of time-dependent GIWO is

$$\left\{\frac{\partial}{\partial t} + (\mathbf{v} + \Delta\tilde{\mathbf{v}})\cdot\frac{\partial}{\partial \mathbf{r}} + \frac{q_b}{m_b}\frac{\partial}{\partial \mathbf{v}}\cdot\left(\tilde{\mathbf{E}} + (\mathbf{v} + \Delta\tilde{\mathbf{v}})\times\tilde{\mathbf{B}}\right)\right\}W(\mathbf{r},\boldsymbol{k},t) = 0 \quad (2)$$

Naturally there are other ways to obtain GIWFs, for example, through certain path integrals involving the vector potential [22]. The main challenge in the use of this approach is in the choice of integration path. However, the phase factor in Eq. (1) can be justified in terms of the minimal coupling principle [14]. Moreover, as discussed in more detail elsewhere, the function of the phase factor is to convert any gauge into the axial gauge [23]. For the purposes of this study, the choice of the form (1) due to convenience as it provides a non-ambiguous way to calculate averaged quantities.

We can compute the very basic zeroth order moment in the velocity space as

$$\int d^3r\, W(\mathbf{r},B) = \psi^*(\mathbf{B})\psi(\mathbf{B}) = |\psi(\mathbf{B})|^2 \quad (3)$$

The paper has been arranged as follows: The presentation starts with theoretical formulations in section 2.1 where we deduce the expression for diamagnetic flow of BFPC based on the electromagnetic interactions using the Wigner operator and the Wigner function. In the next section, 2.2, we use the equation of diamagnetic flow to derive an equation for the vortex radius as a function of particle excitation energy. The well-known formulae that link penetration depth to the vortex radius and the lower critical field are also introduced in section 2.3, to solve the equation of radius of the vortex. Section 3 outlines the discussion on the derived quantities and conclusion is made in section 4.

2. Theoretical Formulations

From the Schrodinger equation, the Cooper-pair energy is defined by

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \quad (4)$$

For a Cooper pair interacting with a magnetic field in the vortex state (time independent) $i\hbar\frac{\partial}{\partial t}$ $\psi = 0$, and V is the magnetic potential energy $\left(\frac{\tilde{\mathbf{B}}\cdot\tilde{\mathbf{B}}}{2N\mu_0}\right)$ per Cooper pair, where N is the number density of static Cooper pairs. Note that the filed $\tilde{\mathbf{B}} = \tilde{\mathbf{B}}_{\boldsymbol{x}} + \tilde{\mathbf{B}}_{\boldsymbol{y}} + \tilde{\mathbf{B}}_{\boldsymbol{z}}$ where $\tilde{\mathbf{B}}_{\boldsymbol{x}} = \tilde{\mathbf{B}}_{\boldsymbol{r}}$ is along the radius of the vortex, $\tilde{\mathbf{B}}_{\boldsymbol{y}}$ is along the interface between the static Cooper pairs and the vortex core and $\tilde{\mathbf{B}}_{\boldsymbol{z}}$ is along the vortex core (along the applied magnetic field). Considering that $i\hbar\nabla$ is the momentum, then

$$-\frac{\hbar^2}{2m_b}\nabla^2 = \frac{\tilde{\mathbf{B}}_r\cdot\tilde{\mathbf{B}}_{\boldsymbol{r}}}{2N\mu_0}$$

Or,

$$-i\hbar\nabla = \left(\frac{m_b}{N\mu_0}\right)^{\frac{1}{2}} \tilde{\mathbf{B}}_{\boldsymbol{r}} \tag{5}$$

Its then obvious that the magnetic field operator along the radius is obtained as

$$\tilde{\mathbf{B}}_{\boldsymbol{r}} = \left(\frac{N\mu_0\hbar^2}{m_b}\right)^{\frac{1}{2}} \nabla \tag{6}$$

And the velocity with quantum corrections in GIWF becomes,

$$\mathbf{v} + \tilde{\mathbf{v}} = -\frac{i\hbar\nabla}{m_b} = \left(\frac{1}{N\mu_0 m_b}\right)^{\frac{1}{2}} \tilde{\mathbf{B}}_{\boldsymbol{r}} \tag{7}$$

Accordingly, the quantum momentum $i\hbar\nabla$ takes the place of $m\mathbf{v}$ in the GIWF while $\int_{-1/2}^{1/2} d\tau \mathbf{A}(\mathbf{r} + \boldsymbol{\tau}\mathbf{s}, t)$ vanishes under time independence because the time lag $\tau = 0$. The wave function $\psi\left(\mathbf{r} + \frac{1}{2}\mathbf{s}\right)$ is also time independent. In addition, $\mathbf{B}_c$ is a classical field expressed as

$$\frac{\boldsymbol{k}^2}{2m} = \frac{\mathbf{B}_c \cdot \mathbf{B}_c}{2N\mu_0} \Rightarrow \mathbf{B}_c = \left(\frac{N\mu_0}{m_b}\right)^{\frac{1}{2}} \boldsymbol{k} \tag{8}$$

And hence,

$$m\mathbf{v} = \boldsymbol{k} = \frac{\boldsymbol{k}}{m_b} = \left(\frac{m_b}{N\mu_0}\right)^{\frac{1}{2}} \mathbf{B}_{\boldsymbol{c}} \tag{9}$$

Thus, the time independent GIWF for a Cooper pair interacting with a magnetic field becomes

$$W(\mathbf{r}, B) = \left(\frac{1}{2\pi\hbar}\right)^3 \int d^3\mathbf{s} \exp\left[\frac{i}{\hbar}\mathbf{s} \cdot \left(\frac{m_b}{N\mu_0}\right)^{\frac{1}{2}} \tilde{\mathbf{B}}_c\right] \times \psi^*\left(\mathbf{r} + \frac{1}{2}\mathbf{s}\right)\psi\left(\mathbf{r} - \frac{1}{2}\mathbf{s}\right) \tag{10}$$

Here, $\hbar = h/2\pi$ and m_b are the reduced Planck's constant and the mass of a Cooper pair in a state described by a wave function ψ. When the bulk of a superconductor is in a pure superconducting state, the time-independent GIWF of a Cooper-pair vanishes since $\mathbf{B}=0$. The complete equation of motion of a boson (Cooper pair) in a magnetic field in form of time-independent GIWO is

$$\left\{\left(\frac{1}{N\mu_0 m_b}\right)^{\frac{1}{2}}\tilde{\mathbf{B}}_r\cdot\frac{\partial}{\partial\mathbf{r}}+q_b\left(\frac{N\mu_0}{m_b}\right)^{\frac{1}{2}}\frac{\partial}{\partial\mathbf{B}_c}\cdot\left(\left(\frac{1}{N\mu_0 m_b}\right)^{\frac{1}{2}}\tilde{\mathbf{B}}_r\times\tilde{\mathbf{B}}_z\right)\right\}\widehat{W}(\mathbf{r},B)=0 \qquad (11)$$

Simplifying leads to

$$\left\{\left(\frac{m_b}{N\mu_0 q_b^2}\right)^{\frac{1}{2}}\tilde{\mathbf{B}}_r\cdot\frac{\partial}{\partial\mathbf{r}}+\frac{\partial}{\partial\mathbf{B}_c}\cdot\left(\tilde{\mathbf{B}}_r\times\tilde{\mathbf{B}}_z\right)\right\}\widehat{W}(\mathbf{r},B)=0 \qquad (12)$$

Multiplying through Eq. (12) by $m_b^{\frac{1}{2}}$,

$$\left\{\left(\frac{m_b^2}{N\mu_0 q_b^2}\right)^{\frac{1}{2}}\tilde{\mathbf{B}}_r\cdot\frac{\partial}{\partial\mathbf{r}}+m_b^{\frac{1}{2}}\frac{\partial}{\partial\mathbf{B}_c}\cdot\left(\tilde{\mathbf{B}}_r\times\tilde{\mathbf{B}}_z\right)\right\}\widehat{W}(\mathbf{r},B)=0 \qquad (13)$$

Where $\widehat{W}(\mathbf{r},B)$ is the Wigner operator. If $\hat{\rho}(B_0)$ is the density operator for quantized degrees of freedom for the system particle and field, the GIWF can be obtained as a trace of the expectation value of GIWO using the relation

$$W=\mathrm{Tr}\left[\widehat{W}\hat{\rho}\right] \qquad (14)$$

Thus, if we multiply Eq. (13) by $\hat{\rho}$ and find its trace then we obtain [19]

$$\left\{\left(\frac{m_b^2}{N\mu_0 q_b^2}\right)^{\frac{1}{2}}\tilde{\mathbf{B}}_r\cdot\frac{\partial}{\partial\mathbf{r}}+m_b^{\frac{1}{2}}\frac{\partial}{\partial\mathbf{B}_c}\cdot\left(\tilde{\mathbf{B}}_r\times\tilde{\mathbf{B}}_z\right)\right\}W(\mathbf{r},B)=0 \qquad (15)$$

The quantum nature of $W(\mathbf{r},\mathbf{v},t)$ is in its dependence on $\hbar$. For instance, in the limit $\hbar\to 0$, $\tilde{\mathbf{B}}\to 0$ and $W(\mathbf{r},\mathbf{v},t)$ vanishes. Therefore, Eq. (12) is only feasible in the vortex state on condition that $\hbar>0$.

A fluid moments hierarchy has derived from the electrostatic Wigner function [26]. In moments theories [18], a set of macroscopic variables (particle density, current etc.) were defined in terms of integrals of the Wigner function. These results were extended to the time-dependent electromagnetic phenomenon [20]. In the case of time independence, we defined

$$\text{Number density of Particles:} \qquad N = \int d^3B\, W \tag{16}$$

$$\text{Magnetic Potential per unit Volume:} \quad \tilde{\mathbf{B}}_\perp = \frac{1}{N}\int d^3B\, W\tilde{\mathbf{B}}_\perp \tag{17}$$

$$\text{Pressure Tensor:} \qquad \mathbf{P} = \left(\frac{1}{2\mu_0}\right)\int d^3B\, W\left(\tilde{\mathbf{B}}_r \cdot \tilde{\mathbf{B}}_r\right) \tag{18}$$

The expectation value of an operator $\langle \hat{G} \rangle = \int d^3B(g(B))$ is a phase-space average of the Wigner function of that operator. Thus, the equation of conservation of $g(B)$ is

$$\int d^3B g(B)\left\{\left(\frac{m_b^2}{N\mu_0 q_b^2}\right)^{\frac{1}{2}}\tilde{\mathbf{B}}_r \cdot \frac{\partial}{\partial \mathbf{r}} + m_b^{\frac{1}{2}}\frac{\partial}{\partial \mathbf{B}_c} \cdot \left(\tilde{\mathbf{B}}_r \times \tilde{\mathbf{B}}_z\right)\right\}W = 0 \tag{19}$$

For a dipole moment, $g(\tilde{\mathbf{B}}) = \tilde{\boldsymbol{\mathcal{M}}} = \frac{1}{\mu_0}\tilde{\mathbf{B}}$ and its expectation value is

$$\langle \hat{G} \rangle = \int d^3B\left(\frac{1}{\mu_0}\tilde{\mathbf{B}}\right) = \frac{1}{\mu_0}\int d^3B\left(\tilde{\mathbf{B}}_{\boldsymbol{x}} + \tilde{\mathbf{B}}_{\boldsymbol{y}} + \tilde{\mathbf{B}}_{\boldsymbol{z}}\right) \tag{20}$$

Thus, the equation of conservation of the dipole moment is

$$\int d^3B\left(\frac{1}{\mu_0}\tilde{\mathbf{B}}\right) \cdot \left\{\left(\frac{m_b^2}{N\mu_0 q_b^2}\right)^{\frac{1}{2}}\tilde{\mathbf{B}}_r \cdot \frac{\partial}{\partial \mathbf{r}} + m_b^{\frac{1}{2}}\frac{\partial}{\partial \mathbf{B}_c} \cdot \left(\tilde{\mathbf{B}}_r \times \tilde{\mathbf{B}}_z\right)\right\}W(\mathbf{r},B) = 0 \tag{21}$$

Each part of Eq. (21) is tackled separately using the commutation property and the moments in Eqs. (16) – (18):

First term of Eq. (21):

$$\int d^3B\left(\frac{1}{\mu_0}\tilde{\mathbf{B}}\right)\cdot\left(\frac{m_b^2}{N\mu_0 q_b^2}\right)^{\frac{1}{2}}\tilde{\mathbf{B}}_r\frac{\partial}{\partial\mathbf{r}}=\left(\frac{m_b}{N\mu_0}\right)^{\frac{1}{2}}\frac{\partial}{\partial\mathbf{r}}\int d^3B\frac{1}{\mu_0}\left(\tilde{\mathbf{B}}_r\cdot\tilde{\mathbf{B}}_r\right)=\left(\frac{m_b^2}{N\mu_0 q_b^2}\right)^{\frac{1}{2}}\nabla\cdot\mathbf{P} \tag{22}$$

Thus, the second term of Eq. (20) is simplified as

$$\int d^3B\left(\frac{1}{\mu_0}\tilde{\mathbf{B}}\right)\cdot\left(m_b^{\frac{1}{2}}\frac{\partial}{\partial\mathbf{B}_c}\cdot\left(\tilde{\mathbf{B}}_r\times\tilde{\mathbf{B}}_z\right)\right)W(\mathbf{r},B)=m_b^{\frac{1}{2}}\frac{\partial}{\partial\mathbf{B}_c}\tilde{\mathbf{B}}_y\cdot\left(\tilde{\mathbf{B}}_r\times\tilde{\mathbf{B}}_z\right) \tag{23}$$

Substituting the solutions back to Eq. (20) we have

$$\left(\frac{m_b^2}{N\mu_0 q_b^2}\right)^{\frac{1}{2}}\nabla\cdot\mathbf{P}+m_b^{\frac{1}{2}}\frac{\partial}{\partial\mathbf{B}_c}\tilde{\mathbf{B}}_y\cdot\left(\tilde{\mathbf{B}}_r\times\tilde{\mathbf{B}}_z\right)=0 \tag{24}$$

Eq. (23) can be factorized as

$$\left(\frac{m_b^2}{N\mu_0 q_b^2}\right)^{\frac{1}{2}}\nabla\cdot\mathbf{P}+m_b^{\frac{1}{2}}\left(\mu_0 q_b^{\frac{1}{2}}\right)\left(\frac{\partial}{\partial\mathbf{B}_c}\tilde{\mathbf{B}}_y\cdot\left(\tilde{\mathbf{B}}_r\times\tilde{\mathbf{B}}_z\right)\right)=0 \tag{25}$$

Or,

$$\left(\frac{m_b^2}{N\mu_0 q_b^2}\right)^{\frac{1}{2}}\nabla\cdot\mathbf{P}+m_b^{\frac{1}{2}}\left(\frac{\partial}{\partial\mathbf{B}_c}\tilde{\mathbf{B}}_y\cdot\left(\tilde{\mathbf{B}}_r\times\tilde{\mathbf{B}}_z\right)\right)=0 \tag{26}$$

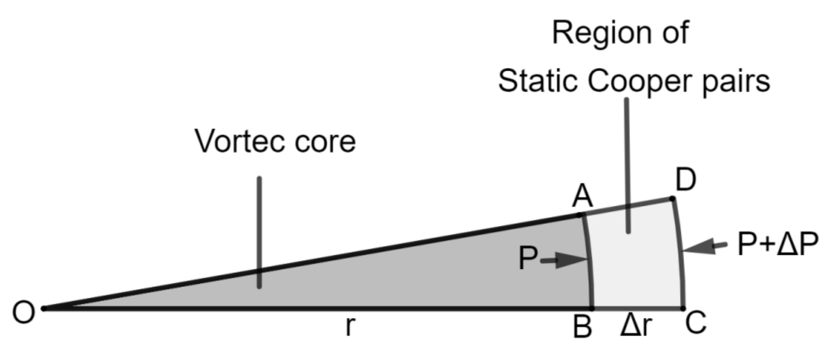
Considering pressure along the radius, we have $\nabla\cdot\mathbf{P}=\frac{\partial P_r}{\partial r}$ and hence

$$\left(\frac{m_b^2}{N\mu_0 q_b^2}\right)^{\frac{1}{2}}\frac{\partial P_r}{\partial r}+m_b^{\frac{1}{2}}\left(\frac{\partial}{\partial \mathbf{B}_c}\tilde{\mathbf{B}}_y\cdot\left(\tilde{\mathbf{B}}_r\times\tilde{\mathbf{B}}_z\right)\right)=0 \qquad (27)$$

We define the first term on the right-hand side of Eq. (30) as the magnetic potential energy that opposes the kinetic energy of the finite-momentum Cooper pairs to attain a static state. Thus,

$$\frac{B^2}{2\mu_0}=\left(\frac{m_b^2}{N\mu_0 q_b^2}\right)^{\frac{1}{2}}\frac{\partial P_r}{\partial r} \qquad (28)$$

In the next few lines, we define $\frac{\partial P_r}{\partial r}$ in terms of the magnetic potential energy. Considering a static fluid element whose radius from the centre of the vortex is r in which Δr is the radial thickness of the element, ΔA as the area of cross-section of the element and $\Delta\theta$ angle subtended by the element at the centre 0.



We will assume that for a very small element ABCD, the area element on side AB is equal to that on side CD. The various forces acting on the element along the radius are:

i) The force due to the superconducting fluid on side DC of the element along the radius, r, is

$$F_{CP} = (P + \Delta P)\Delta A \ = \left(P + \frac{\partial P}{\partial r}\Delta r\right)\Delta A \tag{29}$$

ii) The force due to the vortex core on the element is given as

$$F_V = P\Delta A \tag{30}$$

iii) The difference between F_{CP} and F_V is the force due to magnetic potential energy in the volume V and so

$$\left(P + \frac{\partial P}{\partial r}\Delta r\right)\Delta A - P\Delta A = \frac{B^2}{2\mu_0} \cdot \frac{\mathrm{V}}{r} \tag{31}$$

By defining the volume of the element as $V = \Delta\mathrm{r}\Delta\mathrm{A}$, we find that

$$\frac{\partial P}{\partial r} = \frac{B^2}{2\mu_0} \cdot \frac{1}{r} \tag{32}$$

Substituting Eq. (32) into Eq. (28), we have

$$r = \left(\frac{m_e^2}{N\mu_0 q_e^2}\right)^{\frac{1}{2}} \tag{34}$$

If we consider a Cooper pair interacting with a magnetic field whose potential energy varies within the range $\left[-\frac{B_{c1}H_{c1}}{2\pi\epsilon_0}, \frac{B_{c1}H_{c1}}{2\pi\epsilon_0}\right]$, then

$$N = \sum_{k^{'}}(1) = 2\int_0^{\frac{B_{c1}H_{c1}}{2\pi\epsilon_0 N}} du = \frac{\mu_0 H_{c1}^2}{\pi\epsilon_0 N} \tag{35}$$

The potential energy has been considered for both species of charges – electrons and holes. Simplifying Eq. (35) yields

$$N = \left(\frac{\mu_0 H_{c1}^2}{\pi\epsilon_0}\right)^{\frac{1}{2}} \tag{36}$$

The magnetic potential energy range of integration $\left[-\frac{B_{c1}H_{c1}}{2\pi\epsilon_0 N}, \frac{B_{c1}H_{c1}}{2\pi\epsilon_0 N}\right]$ is comparable to that of thermal excitation $\left[-\frac{10^3}{q_b}(k_B T_c), \frac{10^3}{q_b}(k_B T_c),\right]$ that was used in determining the excitation energy of a boson-fermion pair condensate [5]. Thus,

$$\frac{\mu_0 H_{c1}^2}{\pi\epsilon_0 N} = 2ƙ_B T_c \tag{37}$$

Where $ƙ_B = \frac{10^3}{q_b} k_B$ is the reduced Boltzmann constant [5] and $q_b = 2q_e$ is the boson charge. Considering magnetization of a single species of charges (electrons only), N is halved and

$$H_{c1}^2 = \frac{\pi\epsilon_0}{\mu_0}\left(\frac{\mu_0 H_{c1}^2}{\pi\epsilon_0}\right)^{\frac{1}{2}}(2ƙ_B T_c) = \left(\frac{\pi\epsilon_0}{\mu_0}\right)^{\frac{1}{2}}\left(\frac{10^3}{q_e} k_B T_c\right) H_{c1} \tag{38}$$

Or,

$$H_{c1} = \left(\frac{\pi\epsilon_0}{\mu_0}\right)^{\frac{1}{2}}\left(\frac{10^3}{q_e} k_B T_c\right) = \frac{10^3}{q_b}\left(\frac{\pi\epsilon_0}{\mu_0}\right)^{\frac{1}{2}} E_k \tag{39}$$

Substituting Eq. (39) into Eq. (36) we get

$$N = \left(\frac{\mu_0}{\pi\epsilon_0}\right)^{\frac{1}{2}}\left(\frac{\pi\epsilon_0}{\mu_0}\right)^{\frac{1}{2}}\left(\frac{10^3}{q_e} k_B T_c\right) = \frac{10^3}{q_b} E_k \tag{40}$$

The radius (r_0) of the vortex from Eq. (34) becomes

$$r_0 = \left(\frac{m_e^2}{\mu_0 q_e^2} \times \frac{q_e}{k_B T_c} \times 10^{-3}\right)^{\frac{1}{2}} = \left(\frac{2m_e^2}{\mu_0 q_e E_k} \times 10^{-3}\right)^{\frac{1}{2}} \tag{41}$$

Where $E_k = 2k_BT_c$ is the thermal excitation energy. Multiplying through Eq. (27) by $\left(\frac{m_b^2}{N\mu_0 q_b^2}\right)^{-\frac{1}{2}}$,

we have

$$N^{\frac{3}{4}}\frac{\partial P_r}{\partial r} + \lambda\left(\frac{\partial}{\partial \mathbf{B}_c}\tilde{\mathbf{B}}_y \cdot \left(\tilde{\mathbf{B}}_r \times \tilde{\mathbf{B}}_z\right)\right) = 0 \tag{42}$$

Where,

$$\lambda = \left(\frac{4\mu_0^2 q_e^4}{Nm_e^2}\right)^{\frac{1}{4}} = \left(\frac{8\mu_0^2 q_e^5}{m_e^2 E_k} \times 10^{-3}\right)^{\frac{1}{4}} \tag{43}$$

is the penetration depth. In Eqs. (41) and (43), we estimated that $r_0 \propto m_e$ and $\lambda^4 \propto \frac{1}{N}$ respectively.

3. Results and Discussion

The radius of the vortex core and the penetration depths show dependence on the excitation energy of the Cooper pairs. **Table 1** compares the vortex radii (r_0) in some high temperature superconductors determined from Eq. (37) with the existing empirical findings.

Table 1: *Penetration Depths and Radii* (r_0) *of magnetic vortices in cuprate and iron pnictide superconductors*

Superconductor	$\boldsymbol{T_c}$**(K)**	$r_0(\boldsymbol{nm})$	
		Model Values	**Empirical values**
$YBa_2Cu_3O_7$	92	1.80	2.0 [27]
$Bi_2Sr_2CaCu_2O_8$	95	1.77	2.2 ± 0.3 [28]
$Ba(Fe_{1-x}Co_x)_2As_2$	24	3.52	-
$CaKFe_4As_4$	34	2.96	-

In Y123 and Bi2212, the model values compare closely with the empirical values. Generally, the model values are in close proximity with the empirical range. The size of the vortex is defined by the energy $(B_{c1}H_{c1})$ of static Cooper pairs that is constant at any external field H. The static Cooper pairs shield the superconducting particles in the halo region from direct contact with magnetic field pressure difference between the superconducting particles and the magnetized Cooper pairs.

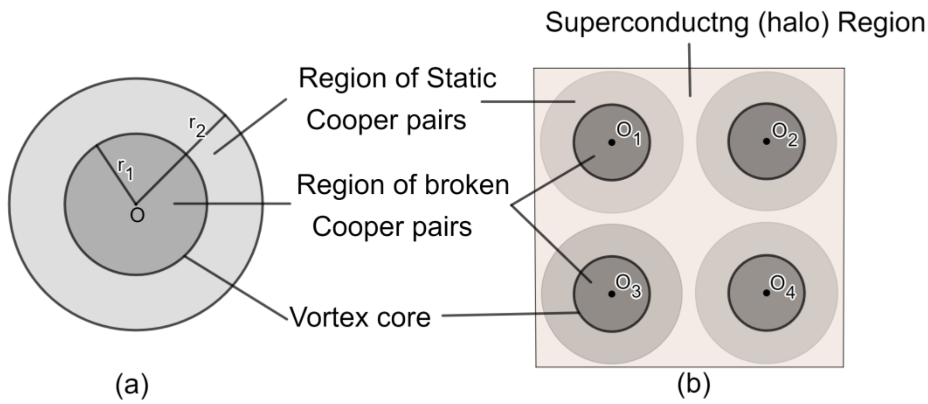


Figure 1: (a) Sections of the vortex core (b) Section of the bulk of aa superconductors. The region of static Cooper pairs shields the superconducting particles in the halo region from direct contact with magnetic fields. However, the magnetic energy is transmitted through density waves (DW) to suppress superconductivity by inhibiting superconducting coherence in the halo region. The region of static Cooper pairs contains a constant magnetic potential energy $(B_{c1}H_{c1})$ and cannot vanish in the superconducting state of a material under any circumstances.

A layer of static Cooper pairs provides for finite momentum Cooper pairs deep in the halo region to interact and pair up with free electrons to form a boson-fermion pair condensate (BFPC) [5] free from magnetic influence. The BFPC model has been discussed in details by Mukubwa and Makokha [5] in which we described its evolution and association with superconductivity and the superconducting energy gap.

The cooperative nature between the components of magnetization $\boldsymbol{B}$ enhances field penetration into the superconducting region of the bulk. **Table 2** shows a summary of magnetic penetration depths in layered superconductors, which has been determined using equation (52).

Table 2: *Penetration Depths* (λ) *and Lower Critical Field* (H_{c1}) *in Cuprate and iron-based superconductors*

Superconductor	T_c(K)	$\lambda(nm)$	
		Model Values	**Empirical values**
$YBa_2Cu_3O_7$	92	158	156 ± 8 [27]
$Bi_2Sr_2CaCu_2O_8$	95	157	269 ± 15 [30]
$Ba(Fe_{1-x}Co_x)_2As_2$	24	222	226 ± 10 [27]
$CaKFe_4As_4$	34	203	208 [31]

The second term of Eq. (42) combines all the three components of the magnetization $\boldsymbol{B}$. The co-operation between the various components of the magnetization $\boldsymbol{B}$ through cross and dot products leads to penetration of the field into the superconducting region. The effect is directed along the vortex boundary. The resulting penetration depth is in agreement with empirical findings (see **Table 2**).

4.Conclusion

Dependence of electromagnetic properties on the excitation of quasi-particles has been derived. Other than the quasi-particle excitation, the properties show dependence on the mass and charge of the Cooper pairs. The radius of vortex is defined by the pressure exerted by the static pairs on the vortex. A layer of static Cooper pairs shields the superconducting particles in the halo region from direct contact with magnetic fields. However, the fields suppress superconductivity in the halo region through charge density waves, which inhibit superconducting coherence. On the other hand, the penetration depths rides on the cooperation between the components of the magnetization $\boldsymbol{B}$.

References

[1] J.O. Odhiambo, T.W. Sakwa, Y.K. Ayodo, B.W. Rapando, Thermodynamic properties of Mercury based cuprate due to Cooper pair - electron interaction, Journal of Multidisciplinary Engineering Science and Technology 3 (7) (2016) 5241–5248.

[2] W.-L. Tu, T.-K. Lee, Evolution of pairing orders between pseudogap and superconducting phases of cuprate superconductors, Sci. Rep. 9 (2018) 1719.

[3] V.V. Tolmachev, Superconducting bose–einstein condensates of cooper pairs interacting with electrons, Phys. Lett. 266 (4–6) (2000) 400–408.

[4] J. Brackett, J. Newman, T.N. De Silva, An effective mean-field theory for coexistence of anti-ferromagnetism and superconductivity: applications to iron-based superconductors and bose-Fermi atomic mixtures, Phys. Lett. 380 (41) (2016) 3421–3429.

[5] Mukubwa, A. and Makokha, J. W. (2021). Energy of plasmon-mediated boson-fermion pair condensate in high temperature superconductors. *Physica B* **618,** 413182

[6] T. Mamedov, M. de Llano, Generalized superconducting gap in an anisotropic boson–fermion mixture with a uniform Coulomb field, J. Phys. Soc. Jpn. 80 (2011), 074718.

[7] R. Cote, A. Griffin, Cooper-pair-condensate Fluctuations and plasmons in layered superconductors, Phys. Rev. B 48 (14) (1993) 10404–10425.

[8] M. Casas, M. de Llano, A. Puente, A. Rigo, M.A. Solis, Pre-formed Cooper pairs and Bose-Einstein condensation in cuprate superconductors, J. Phys. Chem. Solid. 63 (12) (2002) 2365–2368.

[9] T. Dubouchet, B. Sacepe, J. Seidemann, D. Shahar, M. Sanquer, C. Chapelier, Collective energy gap of preformed Cooper pairs in disordered superconductors, Nat. Phys. 15 (2019) 233–236.

[10] Hepting M *et al.* (2018). Three-dimensional collective charge excitations in electron-doped copper oxide superconductors. *Nature* **563,** 374–378.

[11] Nag A., Zhu, M., Bejas, M., Li, J., Robarts, H., Yamase, H., Petsch, A. N., Song, D., Eisaki, H., Walters, A., Garcia-Fernandez, M., Greco, A., Hayden, S. M. and Zhou, K. (2018). Detection of acoustic plasmons in hole-doped lanthanum and bismuth cuprate superconductors using resonant inelastic x-ray scattering, *Physical Review Letters* **125,** 257002

[12] Knupfer, M., Roth, G., Fink, J., Karpinski, J. and Kaldis, E. (1994). Plasmon dispersion and dielectric function in $YBa_2Cu_4O_8$ single crystals. *Physica C* **230,** 1 – 2

[13] Ishii, K. *et al.* (2017). Observation of momentum dependent charge excitations in a hole-doped cuprates using resonant inelastic X-ray scattering at the oxygen K edge. *Physical Review B* **96,** 115148.

[14] Greco, A., Yamase, H. and Beja, M. (2019). Origin of high-energy charge excitations observed by resonant inelastic X-ray scattering in Cuprates. *Communications Physics* **2,** 3.

[15] Varshney, D., Shah, S. and Singh, R.K. (2000). Specific heat studies in Ho–Ba–CuO superconductors: Fermionic and bosonic contributions. *Bulletin of Material Science*, **23(4):** 267 – 272

[16] Weger, M. and Burlachkov, L. (1994). Reduced Screening at the Fermi surface of high-T_c cuprates. *Physica C* **235 – 240,** 2387 – 2388

[17] Konshelle, F. (2014). Transport equation for superconductors in the presence of spin interactions. *The European Physical Journal* **87,** 119

[18] Tokatly, V. (2017). Usadel equation in the presence of intrinsic spin-orbit coupling: A unified theory of magneto-electric effects in normal and superconducting system. *Physical Review* B **96,** 060502

[19] Serimaa, O. T., Javanainen, J. and Varro, S. (1986). Gauge-independent Wigner function: General formulation. *Physical Review A* **33(5),** 2913 – 2927

[20] Haas, F., Marklund, M., Zamanian, J. and Brodin, G. (2010). Fluid moment hierarchy equations derived from gauge invariant quantum kinetic theory. *New Journal of physics* **12,** 073027

[21] Bialynicki-Birula, I., Gornicki, P., and Rafelski, J. (1991). *Physical Review D***44,** 1825

[22] Carruthers, P. and Zachariasen, F. (1983). *Review of Modern Physics* **55,** 245

[23] Levanda, M. and Fleurov, V. (2001). *Annals of Physics* **292,** 199

[24] Fujita, S., *Introduction to non-equilibriumQuantum Statistical Mechanics* (Saunders, Philadelphia, 1966

[25] Bialinicki-Birula, I., Acta Phys. Austriaca, Suppl. XVIII, 111 (1977).

[26] Haas, F., Marklund, M., Brodin, G. and Zamanian, J. (2010). From extended phase-space to fluid theory. *Physics Letters A* **374,** 481.

[27] Mourachkine, A. (2004). Determination of the Coherence Length and the Cooper-Pair Size in Unconventional Superconductors by Tunneling Spectroscopy. *Journal of Superconductivity* **17,** 711–724

[28] Sonier, J.E., Brewer, J.H. and Kiefl, R.F. (2000). μSR Studies of the Vortex State in Type-II Superconductors. *Reviews of Modern Physics* **72,** 769

[29] Mukubwa, A. and Masinde, F. (2019). Determining the Radius of the Magnetic Vortex Core of YBCO123 and Bi2212. *Open Access Library Journal* **6,** e5661.

[30] Joshi, K. R., Nusran, N. M., Cho, K., Tanatar, M. A., Meier, W. R., Bud'ko, S. L., Canfield, P. C. and Prozorov, R. (2019). Measurements of the lower critical field of superconductors using nitrogen vacancy centers in diamond optical magnetometry. *Physical Review Applied* **11(1),** 014035

[31] Prozorov, R., Giannetta, R. W., Carrington, A., Fournier, P., Greene, R. L., Guptasarma, P., Hinks, D. G., and Banks, A. R. (2000). Measurements of the absolute value of the penetration depth in high-T_c superconductors using a low-Tc superconductive coating," *Applied Physics Letters* **77,** 4202-4204.