# Forensic Estimation of Time of Death: A Mathematical Model

[1]Boniface O. Kwach, [2]Omolo_Ongati, [3]Nyakinda J. O, [4]Rachael Nyang'inja
[1,2,3]School of Mathematics and Actuarial Science
[4]Department of Mathematics and Computer Studies
[1,2,3]Jaramogi Oginga Odinga University of Science and Technology
[4]Kisumu Polytechnic
[1,2,3]P.O.Box 210, Bondo, Kenya
[3]P.O.Box 143, Kisumu, Kenya
[1]brokwach@yahoo.com, [2]nomoloongati@gmail.com,
[3]jnyankindas@yahoo.com, [4]rachaelowaga@yahoo.co.uk

**Abstract**

In this paper we establish the exact time of death of a murdered person. This leads to an ordinary differential equations whose solution has been analyzed to provide the approximate time of death. Forensic expert will try to estimate this time from body's current temperature and calculating how long it would have taken to lose heat to reach this point. This provides an accurate approach to establish the approximate time when crime is committed.

**Mathematics Subject Classification:** Primary 93B05; Secondary 91B74, 93C95, 92D40
**Keywords:** Forensic Science, Postmortem Interval, Forensic Entomology

# 1 Introduction

In nature's Crime Scene Creatures, we meet a specialized breed of scientists called forensic entomologists who study the insects associated with a human corpse. Though their work may seem less than glamorous to some, they are in fact vital members of criminal investigative teams. Forensic entomology is actually any interaction between the insects and the legal system. This may involve stored product contamination, nuisance problems, structural damage (termites, etc.), and medico-legal, Lee Goff [2].
Entomologist is called to the scene as soon as a body is discovered and determine that entomological evidence is involved. The entomologist goes to the scene and begins the examination and collections there. Typically, a visit to the morgue to make additional collections and observations during the autopsy procedure is done. If the case is on the mainland, they are often contacted while the scene is being assessed and takes the investigators through the process. If the body is moved before collections are made, the entomologist can

lose significant evidence. The insects will leave the body or move to a different part of the body.
Generally, work begins with the upper exposed surfaces of the body and then, as the body is moved, the lower surfaces is looked and then the substrate. The entomologist does not remove or otherwise disturb clothing during these examinations so that other types of evidence will not be disturbed. For the immature species, collections are split into two lots. One lot is fixed and preserved at the scene in order to stop the biological clock. The other is placed into a rearing container to allow these specimens rear to the adult stage in the laboratory for positive species-level identification. Many times the adults are easy to identify but the larvae are indistinguishable. Collections are made from all different areas of the body and surroundings, and these are all kept separate. Collections during the autopsy procedure are made. In the morgue, the entomologist can spend more time going through clothing for species that might have been missed in the field. Examinations of the internal tissues and wounds may also yield species of major significance to the final analysis, Catts [3].
A body has been discovered; there is need to collect the crime scene creatures and analyze the evidence, then determine the postmortem internal (PMI). After taking the specimen back to the forensic lab, where they have reared and analyzed them the developmental rates based on various factors including weather, climate, the temperature of the body, soil and insect masses are worked out .

## 2 Crime Scene Creatures

Adult flies identified us blowfies and houseflies are considered "pioneer" flies as they are attracted to a corpse within minutes and typically the first witness to death. The flies are valuable in identifying species, but just when they arrived on the scene is unknown, Aaron *et al* [1].
Fly eggs identified as the eggs of the blue bottlefly may be laid within the first two hours, Sukontason *et al* [8]. Large Maggots which are typically older and therefore more important than small ones if they are of the same species grow in size with age. In addition to preserving a few in order to "stop" the clock on their development, you have also reared half a dozen in a temperature controlled environment that mirrors that of death site to identify the development rate and species. You have discovered they are species of housefly in their third "instar" or stage of development, which take four days to reach, Mendona and Paloma *et al* [6].
Beetle identified as a rove beetle arrives shortly after the maggot hatch, typically within a day and half, to prey on the larvae, Scott *et al* [7].

Empty fly pupa case found in the hair of the victim; gives a solid link to the body. It gives clue that at least one generation of the fly has completed development and has hatched out from the case. This entire process takes around 15 days.
Scorpion though formidable looking, this desert denizen is no way linked to the body. Primarily nocturnal and sensitive to light, it is most likely just using the body for shelter during the day. Black window spider despite its deadly reputation, this venomous spider rarely infects enough toxins to kill a person. Though if it is found on a body this spider has no connection with the victim. It may simply be wondering in search of food. Fly with cramped wings is identified as having just emerged from its puparium and hasn't gained the mobility to fly yet. Its proximity to the body links it to the victim. You have determined that it takes 16 days from the time eggs were laid to reach the stage, Whitworth *et al* [9].
This is a great introduction to forensics and using Entomology to solve crimes. It is very good for the educational service it is providing. It's simply meant to inform us that with the help of insects we can learn something important about death, murder and homicide, Greenberg *et al* [5]
As insects and invertebrates colonize a corpse in successive and measured rates, oldest specimen that can be linked to the body will provide the best clues. Your skills as detective and scientist are critical in helping the police target their search for a killer based on the estimated time of murder.
A body has been discovered. A fatal gunshot wound to the chest indicates homicide. As a forensic Entomologist, your job is to examine the crime scene, collect and analyze the evidence and determine the PMI (Postmortem Interval), the minimum elapsed time between death and discovery, Interactive: Determine the Time of Death [4].
There are tools you will need to collect the evidence. These are:

**(a)** Trowel:- used to dig up soil samples.

**(b)** Thermometer:- used to take temperature of air and maggot mass. The heat the maggots produce may have an impact on the rate of their development.

**(c)** Hand net:- for collecting insects.

**(d)** Secure, ventilated jars:- Larval need air to be reared in the laboratory to determine their species and development rate. Use these to keep them separate and be sure to note the date, time and location/body part on which the specimen is collected.

**(e)** Forceps:- for collecting crawling insects.

**(f)** Capped specimen jars with 70 percent ethyl Alcohol solution:- Preserve some specimen in order to "stop" their biological clock.

**(g)** Portable weather station:- used to gather weather data for five to seven days to establish pattern.

# 3 A Mathematical Model

The time of death of a murdered person can be determined with the help of modeling through differential equation. A police personnel discovers the body of a dead person presumably murdered and the problem is to estimate the time of death. The body is located in a room that is kept at a constant 70 degree F. For some time after the death, the body will radiate heat into the cooler room, causing the body's temperature to decrease assuming that the victim's temperature was normal 98.6F at the time of death. Forensic expert will try to estimate this time from body's current temperature and calculating how long it would have had to lose heat to reach this point.
Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature, that is the temperature of its surroundings.
Newton's Law makes a statement about an instantaneous rate of change of the temperature. This verbal statement is translated into a differential equation, which we need to solve. The solution to this equation will then be a function that tracks the complete record of the temperature over time. Newton's Law would enable us to solve the time of death of a murdered person.
The rate of change of the temperature $dT/dt$ , is (by Newton's Law of Cooling) proportional to the difference between the temperature of the body $T(t)$ and the ambient temperature $T_s$. This means that:

$$\frac{dT}{dt} \propto (T - T_s).$$

Here a bit of care is needed: Clearly if the body is hotter than the surrounding temperature $T(t) - T_s > 0$ , then the body is cooling down and the derivative $dT/dt$ should be negative. The equation we need has to have the following sign pattern:

$$\frac{dT}{dt} = -k(T - T_s).$$

where $k$ is a positive constant. The independent variable is $t$ for time, the function we want to find is $T(t)$ , the quantities $T_s$ and $k$ are constants. We define;

$$y(t) = T(t) - T_s$$

as temperature difference between body and the surrounding at time t and

$$y_o = T(0) - T_s = T_o - T_s$$

as initial temperature difference at time t=0
Note that if we take a derivative of $y(t)$ and use the Newton's law of cooling, we arrive at

$$\frac{dy}{dt} = \frac{d}{dt}(T(t) - T_s) = \frac{dT}{dt} - \frac{dT_s}{dt} = \frac{dT}{dt} = -k(T - T_s) = -ky$$

We have used the fact that $T_s$ is constant to eliminate its derivative, and we substitute in $y$ for $(T - T_s)$ in the last step. By defining this new variable, we have arrived once more at the familiar equation

$$\frac{dy}{dt} = -ky$$

whose solution is;

$$y(t) = y_o e^{-kt}$$

We can use this result to conclude by substituting in $y = (T - T_s)$ and $(T_o - T_s)$ that

$$T(t) - T_s = (T_o - T_s)e^{-kt}$$

It follows that

$$T(t) = T_s + (T_o - T_s)e^{-kt}$$

Newtons Law of Cooling states that:

$$T(t) = T_s + (T_0 - T_S)e^{-kt}$$

where;
t is the time in the preferred units (seconds, minutes, hours, etc.)
T(t) is the temperature of the object at time t
$T_s$ is the surrounding constant temperature (ambient temperature)
$T_0$ is the initial temperature of the object
k is a constant to be found.
If $T(t)$ is the body temperature at time t, then for some constant of proportionality $k$,

$$T^{'}(t) = k[T(t) - 70]$$

This is a separable differential equation and is written as

$$\frac{1}{T-70}dT = kdt$$

Upon integrating both sides, one gets

$$ln|T-70| = kt + c$$

Taking exponential, one gets

$$|T-70| = e^{kt+c} = Ae^{kt}$$

where $A = e^c$.
Then;

$$T - 70 = Ae^{kt} = Be^{kt}$$

Then;

$$T(t) = 70 + Be^{kt}$$

Constants $k$ and $B$ can be determined provided the following information is available:

- time of arrival of the police personnel
- the temperature of the body just after his arrival
- temperature of the body after certain interval of time

Suppose the officer arrived at $10.40p.m.$ and the body temperature was 94.4 degrees. This means that if the officer considers $10:40p.m.$ as $t = 0$ then $T(0) = 94.4 = 70 + B$ and so $B = 24.4$ giving

$$T(t) = 70 + 24.4e^{kt}$$

If the officer makes another measurement of the temperature say after 90 minutes, that is, at $12.10a.m.$ and temperature was 89 degrees. This means that

$$T(90) = 89 = 70 + 24.4e^{90k}$$

Then;

$$e^{90k} = \frac{19}{24.4}$$
$$90k = ln(\frac{19}{24.4})$$

and

$$k = \frac{1}{90} ln(\frac{19}{24.4})$$

The officer has now temperature function

$$T(t) = 70 + 24.4e^{\frac{t}{90} ln(\frac{19}{24.4})}$$

In order to find when the last time the body was 98.6 (presumably the time of death), one has to solve for time the equation;

$$T(t) = 98.6 = 70 + 24.4e^{\frac{t}{90} ln(\frac{19}{24.4})}$$

To do this, the officer writes

$$\frac{28.6}{24.4} = e^{\frac{t}{90} ln \frac{19}{24.4}}$$

and takes logarithms of both sides to obtain

$$ln \frac{28.6}{24.4} = \frac{t}{90} ln \frac{19}{24.4}$$

Therefore, the time of death, according to this mathematical model, was;

$$t = \frac{90 ln(28.6/24.4)}{ln(19/24.4)}$$

which is approximately $-57.0.7$ minutes.
The death occurred approximately 57.07 minutes before the first measurement at $10.40 p.m$, that is at $9.43 p.m$.

# 4 Conclusion

The most common misconception with regard to forensic entomology is the level of precision of our results. They provide estimates of a period of insect activity on the body and not the actual period of time since death. These are often quite similar, but not always. They begin with ranges in hours, then days, months, season, and, finally, "It's been there a long time." This paper provides an accurate approach to establish the approximate time when the crime was committed.

## Acknowledgements:

Thanks to Dr. Amos Otedo for his valuable comments on one of our discussions and allowing us to access the secondary data.

## References

[1] **Aaron M. Tarone., Kimberley C. Jennings., David R. Foran, (2007)**. Aging Blow Fly Eggs Using Gene Expression: *J Forensic Sci*, Vol. 52, No. 6

[2] **Goff M. L, (2000)**. *A Fly for the Prosecution*, Cambridge, Massachusetts: Harvard University Press.

[3] **Catts E. P., Haskell N. H,(1990)**. *Entomology and Death: A Procedural Guide*, Joyce's Print Shop, Inc. p5.

[4] **Interactive, (2011)**. Determine the Time of Death. http://www.pbs.org/wnet/nature/episodes/crime-scene-creatures/interactive-determine-the-time-of-death/4390/

[5] **Greenberg, Bernard, John C. K, (2002)**. *Entomology and the Law*, United Kingdom: Cambridge University Press.

[6] **Mendona, Paloma Martins, (2008)**. Identification of fly eggs using scanning electron microscopy for forensic investigations; *Micron*, 13 March 2008.

[7] **Scott, Michelle Pellissier, (1998)**. The ecology and behavior of burying beetles, *Annual Review of Entomology* 43: 595618. doi:10.1146/*annurev.ento*.43.1.595. *PMID* 15012399

[8] **Sukontason, K. L, Piangjai, Boonchu S. N; Kurahashi, Hope H, Olson M. JK (2004)**. Identification of Forensically Important Fly Eggs Using A Potassium Permanganate Staining Technique, *Micron* 35 (5): 391395. doi:10.1016/*j.micron*.2003.12.004. *PMID* 15006363.

[9] **Whitworth, Terry, (2006)**. Keys to Genera and Species of Blow Flies of America North of Mexico, *Proceedings of the Entomological Society of Washington* 108 (3): 710.