(Knowledge for Development)

KIBABII UNIVERSITY

VUNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAP 313

COURSE TITLE: GROUP THEORY I

DATE: 05 /12/2023 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a. Define the following
   i. Abelian group (2mark)
   ii. Subgroup (3marks)
   iii. Left coset (2marks)

b. Let G be a group and $a, b \in G$. Show that the equation $ax = b$ has a unique solution (5 marks)

c. Let G be a group. Show that $x * z = y * z \Rightarrow x = y$ for $x, y \in G$ (4 marks)

d. Let K be the subgroup of s_3 defined by the permutations $\{(1), (12)\}$ determine the right cosets of K in S_3 (6marks)

e. Let H be a subgroup of a group G. Show that the left cosets of H in G partition G. (5marks)

f. Show that every subgroup H of an abelian group G is normal (3marks)

QUESTION TWO

a. Let $\varphi: G \rightarrow H$ be a homomorphism, and let e, $e^{/}$ denote the identity elements of G and H respectively. Show that
   $i.\ \varphi(e) = e^{/}$ (1mark)
   $ii.\ \varphi(a^{-1}) = \varphi(a)^{-1}$ (1mark)
   $iii.\ \varphi(a^{n}) = \varphi(a)^{n}$ (3marks)

b. Show that φ is a monomorphism if and only if $\ker \varphi = \{e\}$ (6marks)

c. Given $g \in G$ define the map $\varphi: G \rightarrow G$ by $\varphi(a) = gag^{-1}$ Show that $\varphi \in AutG$. (4marks)

d. Define the following
   i. Permutation (1mk)
   ii. Symmetric group (2mks)
   iii. Alternating group (2mks)

QUESTION THREE (20 MARKS)

a. Define the following
   i. Center of a group (2mks)
   ii. Homomorphism (2mks)

b. The center Z of a group G is a normal subgroup of G (7mks)

c. If Φ:G→H is a homomorphism then Im (Φ) $\cong {}^{G}/_{Ker(\Phi)}$ (9mks)

QUESTION FOUR (20 MARKS)

a. Define the following
   i. Normal subgroup (1mark)
   ii. Simple group (1mark)
   iii. Composition series (2marks)
   iv. Cyclic group (2marks)

b. Show that every cyclic group is abelian (4 marks)

c. Show that every subgroup of a cyclic group is cyclic (7marks)

d. Let G be a group. Show that $r * t = l * t \Rightarrow r = l$ for , $l \in G$ (4 marks)

QUESTION FIVE (20 MARKS)

a. Define the following
   i. Group action (3marks)
   ii. Orbit (2marks)
   iii. Stabilizer (2marks)

b. Show that stab(x) is a subgroup of G for each $x \in X$ (5marks)

c. Show that the orbits of an action partition X (4marks)

d. Show that if $|G| = n$, then there is an embedding $G \rightarrow S_n$ (4 marks)