(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE: **MAP 121**

COURSE TITLE: **ALGEBRAIC STRUCTURES I**

DATE: 25/4/2023 **TIME: 2:00 PM – 4:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following

i. Binary operation (2marks)

ii. Group (4marks)

iii. Composition of functions (2marks)

b) Prove that the identity element of a group is always unique (4marks)

c) Describe the Dihedral group D_5 (8marks)

d) Let G be a group and $H \trianglelefteq G$ prove that the the set ${}^{G}/_{H}$ forms a group (6marks)

e) State four examples of groups (4marks)

QUESTION TWO (20 MARKS)

a) Define the following

i. Subgroup (3marks)

ii. Klein-four group (2marks)

iii. Alternating group (2marks)

b) State the properties of a ring (3marks)

c) State the distinct left cosets of $\langle 4 \rangle$ in $\mathbb{Z}_{12}$ (5marks)

d) Prove that a finite group G whose order is a prime p is cyclic (5marks)

QUESTION THREE (20 MARKS)

a) Define the following

i. Ring (5marks)

ii. Normal subgroup (2marks)

iii. Quarternion group (2marks)

b) Given the set $S_3 \geq < (13) >$, state the distinct cosets of $< (13) >$ in S_3 (5marks)

c) Prove that every subgroup of an abelian group is normal (4marks)

d) State the properties of a Field (2marks)

QUESTION FOUR (20 MARKS)

a) Find the inverse of the following matrix, whose entries are elements of Z_6 (6marks)

$$A = \begin{bmatrix} 1 & 2 \\ 5 & 5 \end{bmatrix}$$

b) State the Lagrange's theorem (2marks)

c) Construct the Cayley table for the Klein-4 group and prove its abelian (8marks)

d) State 4 examples of binary operations (4marks)

QUESTION FIVE (20 MARKS)

a) Define the following terms

i. Abelian group (2marks)

ii. Proper subgroup (2marks)

iii. Cyclic group (2marks)

iv. Range of a function (2marks)

b) Discuss the symmetric group S_3 and give its subgroups and prove if it satisfies the Lagrange's theorem (12marks)