(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 423/MAA 413**

COURSE TITLE: **ORDINARY DIFFERENTIAL EQUATION II**

DATE: 13/4/2023 **TIME:** 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Determine the stability of the system $\dot{x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} x$ (5 marks)

b) Show that there exist a unique solution to the differential equation

$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$, hence find the unique solution. (7 marks)

c) Linearize and hence solve the non-linear differential equation $y' = y^2 + 1$ at $y(0) = 1$

(6 marks)

d) Use matrix method to solve the following system of differential equations

$$X' = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ (8 marks)

e) Use Picard's method to approximate the value of y when x=0.1 given that y = 1 when x = 0 and $\frac{dy}{dx} = x + y$. (4 marks)

QUESTION TWO (20 MARKS)

a) Use elimination method to solve the system

$$2\frac{dx}{dt} + \frac{dy}{dt} + x - y = 0$$

$$\frac{dx}{dt} + \frac{dy}{dt} + 9x = 9$$

(10 marks)

b) Use row reduction method to solve the differential equation defined by $(x^2 - 1)y^{II} - 2xy^I + 2y = o$ given that $y = x$ is a solution of the differential equation. (10 marks)

QUESTION THREE (20 MARKS)

Find the power series solution for the initial value problem

$xy''+y'+2y=0$

$y(1)=2$
$y'(1)=2$

at the ordinary point $x=1$ (20 Marks)

QUESTION FOUR (20 MARKS)

a) State the condition for the following critical points to occur and in each case draw the phase portrait
   i) Node (2 marks)
   ii) Saddle point (2 marks)

b) Consider a nonlinear system

$$f(x)=\begin{bmatrix} x_1^2 - x_2^2 - 1 \\ 2x_2 \end{bmatrix} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix}$$

Analyze the system by

i) Finding the critical points (4 marks)

ii) Linearize the system and determine the type of critical point it has. Draw the phase portrait in each case. (12 marks)

QUESTION FIVE (20 MARKS)

a) Find the general solution of the system $X' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} x$ (6 marks)

b) Determine the respective fundamental matrix $x(t)$ given that $x(0)=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (11 marks)

c) Hence find $e^{\begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} t}$ (3 marks)