(Knowledge for Development)

KIBABII UNIVERSITY

(KIBU)

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

SPECIAL/SUPPLEMENTARY EXAMINATION

YEAR ONE SEMESTER TWO EXAMINATIONS

FOR THE DEGREE OF BACHELORS OF SCIENCE (COMPUTER SCIENCE)

COURSE CODE : CSC 121

COURSE TITLE : DISCRETE STRUCTURES II

DATE: 10/08/2023 **TIME:** 2.00P.M. - 4.00P.M. **2HRS**

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTIONS ONE AND ANY OTHER TWO.

QUESTION ONE **[30 MARKS]**

a. Represent the proposition $(\sim A \vee B) \rightarrow (C \wedge \sim B)$ by its truth table. **[2 marks]**

b. Write the negation of the statement: $(\exists x)\,(\forall y)\,p(x, y)$. **[2 marks]**

c. What is the probability that when two dice are rolled, the product of the numbers on the two dice is less that 10? **[3 marks]**

d. Write the following statement in symbolic form using quantifiers:

(i) All students have taken a course in logic **[2 marks]**

(ii) Some students are intelligent, but not hardworking **[2 marks]**

e. Solve the following congruence for x. $7x \equiv 12 \pmod{13}$. **[3 marks]**

f. Translate the following sentences into a formula in propositional logic: "If Mr. Holmes told the truth and Mr. Watson did not hear anything, then it cannot be both that the butler did it and that the butler returned to his hotel room that night." **[3 marks]**

g. Determine the solution of the recurrence relation: $F_n = 20F_{n-1} - 25F_{n-2}$ where $F_0 = 4$ and $F_1 = 14$. **[6 marks]**

h. Suppose a fair coin is tossed six times: Find

i. The number of heads which can occur with their respective probabilities **[2 marks]**

ii. The mean or expectation **[2 marks]**

i. Explain the application of **The Chinese remainder theorem** and **Little Fermat's theorem** as used in the study of discrete structures. **[3 marks]**

QUESTION TWO **[20 MARKS]**

a. Find the remainder when 3^{100000} is divided by 53. **[4 marks]**

b. Find the solution of the linear homogeneous recurrence relation of second-degree with constant coefficients $a_n = 7a_{n-1} - 10a_{n-2}$, $a_0 = 2$, $a_1 = 3$. **[4 marks]**

c. Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} - a_{n-2}$ for n = 2 , 3 , 4, . . . , and suppose that $a_0 = 3$ and $a_1 = 5$. What are a_2 and a_3? **[2 marks]**

d. Use mathematical induction to show that $2 + 4 + 6 + \ldots + 2n = n^2 + n$, for $n \geq 1$ **[4 marks]**

e. Find a positive integer ***(a)*** such that when ***(a)*** is divided by **3** it gives a remainder of **2**, when divided by **5** remainder is **4** and when divided by **7** remainder is **6**. **[6 marks]**

QUESTION THREE [20 MARKS]

a. How many edges are there in a graph with 10 vertices each of degree 5? **[3 marks]**

b. Does there exists a simple graph with the degree sequence 3 , 3 , 3 , 2 ? **[3 marks]**

c. Find the number of edges and vertices in $K_{50,100}$ **[3 marks]**

d. Graph G is represented by the following adjacency matrix A

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

i. Draw the graph. **[3 marks]**

ii. Determine whether G is Hamiltonian graph. Justify your answer. **[3 marks]**

iii. Determine whether G is Eulerian graph. Justify your answer. **[3 marks]**

iv. Determine whether G is a tree. Justify your answer **[2 marks]**

QUESTION FOUR [20 MARKS]

a. Let X be a random variable which assigns to each point in S the sum of the numbers; where S consists of 36 ordered pairs of (a, b) where a and b can be any of the integers from 1 to 6. Find the distribution of X. **[4 marks]**

b. A pair of dice is loaded. The probability that a 4 appears on the first die is 2/7, and the probability that a 3 appears on the second die is 2/7. Other outcomes for each die appear with probability 1 /7. What is the probability of 7 appearing as the sum of the numbers when the two dice are rolled? **[4 marks]**

c. What is the probability that the numbers 11,4, 17,39, and 23 are drawn in that order from a bin containing 50 balls labeled with numbers 1,2, . . ., 50 if:

i The ball selected is not returned to the bin before the next ball is selected and; **[3 marks]**

ii The ball selected is returned to the bin before the next ball is selected? **[3 marks]**

a. Let X denote the number of times heads occur when a fair coin is tossed six times.

i. Find the distribution of X (I.e. number of heads which can occur with their respective probabilities) **[2 marks]**

ii. Compute the expectation of X. **[2 marks]**

iii. Compute the variance of X. **[2 marks]**

QUESTION FIVE [20 Marks]

a. Define the following terms:

   i Relatively prime [1 mark]

   ii Modular arithmetic [1 mark]

b. Given as a=365 and b=211 find g(a, b)=s(a) + v(b) [5 marks]

c. Find a positive integer (a) such that when (a) is divided by 7 it gives a remainder of 4, when divided by 9 remainder is 5 and when divided by 11 remainder is 6. [5 marks]

d. Find the remainder when 5^{100000} is divided by 53. [3 marks]

e. Find the least positive values of x such that

   i $84x-38\equiv 79 \pmod{15}$. [3 marks]

   ii $78+x \equiv 3 \pmod{5}$ [2 marks]

f. Using extended Euclidian algorithm find g(a, b)=s(a) + v(b), given a=365 and b=211

   [5 marks]