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(Knowledge for Development)

KIBABII UNIVERSITY

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UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR

MAIN EXAMINATION
YEAR ONE SEMESTER TWO EXAMINATIONS

FOR THE DEGREE OF BACHELORS OF SCIENCE
(COMPUTER SCIENCE)

COURSE CODE : CSC 121

COURSE TITLE : DISCRETE STRUCTURES II

DATE: 25/04/2023 TIME: 2.00 P.M. - 4.00 P.M. 2HRS

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTIONS ONE AND ANY OTHER TWO.

QUESTION ONE (COMPULSORY) [30 MARKS]

a. State the pigeonhole principle and explain its implication. **[3 marks]**

b. Suppose A and B are events with P(A) =0.6, P(B)=0.3 and P(A∩ B)=0.2. find the probability that:

- **i.** A or B occurs **[2 marks]**
- **ii.** Neither A nor B occurs **[2 marks]**

c. Using appropriate notations differential between mod function and mod congruence **[4 marks]**

d. Find the least positive value of x such that:

- **i.** $78+x \equiv 3 \pmod 5$ **[2 marks]**
- **ii.** $5x \equiv 4 \pmod 6$ **[2 marks]**

e. What is First Order Logic (FOL)? How is it related to propositional logic and predicate logic? **[2 marks]**

f. Using relevant examples, differentiate between linear homogeneous recurrences and linear non-homogeneous recurrences. **[2 marks]**

g. Prove that $2^n + 6* 9^n$ is always divisible by 7 for any positive integer n. **[3 marks]**

h. Consider the recurrence relation: $a_n = 4a_{n-1} - 4a_{n-2}$. Find:

- **i.** The general solution **[3 marks]**
- **ii.** The unique solution when initial conditions $a_0=1$ and $a_1=8$. **[3 marks]**
- **iii.** $a_{10}=8$. **[2 marks]**

QUESTION TWO [20 MARKS]

a. Using relevant arguments explain the meaning of symbolic logic. **[2 marks]**

b. *Differentiate between Modus Ponens and Modus Tollens using appropriate examples.* **[2 marks]**

c. Find the values of a_4 for the recurrence relation $a_n = 2a_{n-1} +3$, with $a_0=6$. **[2 marks]**

d. Does there exist a connected graph with degree sequence 1, 1, 1, 1, 2, 2, 2? **[2 marks]**

e. What is the solution of the recurrence relation: $a_n = -a_{n-1} - 4a_{n-2} +4a_{n-3}$ with initial conditions $a0=48$, $a_1 =6$ and $a_2 = 26$. **[6 marks]**

f. Using Chinese Remainder Theorem, find a positive integer(s) (X) such that when it is divided by 11 it gives a remainder 7, when divided by 13 the remainder is 10 and when divided by 15 the remainder is 13. **[6 marks]**

QUESTION THREE [20 MARKS]

a. Define the terms

- **i.** Equiprobable space **[1 mark]**
- **ii.** Expectation **[1 mark]**
- **iii.** Binomial distribution **[1 mark]**
- **iv.** Random variable **[1 mark]**

b. Given E as an event in a sample space with $P(E) > 0$. Find the condition probability of A given E. **[2 marks]**

c. A fair coin is tossed 6 times, call head a success :

- **i.** Find the probability that exactly two success occurs **[3 marks]**
- **ii.** The probability of getting at least four successes. **[3 marks]**
- **iii.** The probability of getting no success. **[2 marks]**

d. Given the distribution below:

X_i	1	3	4	5
P_i	0.4	0.1	0.2	0.3

Find the expectation E(X), the Variance, Var(X) and the standard deviation [Var(X)]1/2 of the distribution. **[6 marks]**

QUESTION FOUR [20 MARKS]

a. What is computation complexity? Differentiate between NP-hard and NP-complete problem **[3 marks]**

b. Explain the condition that a graph should have in order to form a Eurelian path? **[3 marks]**

c. Graph G is represented by the following adjacency matrix A

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

- **i.** Draw the graph. **[3 marks]**
- **ii.** Determine whether G is Hamiltonian graph. Justify your answer. **[3 marks]**
- **iii.** Determine whether G is a tree. Justify your answer **[2 marks]**

d. What is a spanning tree? Using Kruskal's algorithm find the minimum spanning tree for the graph below. **[6 marks]**