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The Specific Heat Of Boson-Fermion Pair Condensate In Optimally-Doped Cuprates

Abel Mukubwa
Department of Science Technology and Engineering
Kibabii University, P.O Box 1699 – 50200,
Bungoma, Kenya.
Tel (Mobile phone): (+254) 717 285 762
e-mail: abelmuwa@gmail.com

John Wanjala Makokha
Department of Science Technology and Engineering
Kibabii University, P.O Box 1699 – 50200,
Bungoma, Kenya.

Fred Wekesa Masinde
Department of Physical Sciences
University of Kabianga, P.O Box 2030 - 2020, Kericho, Kenya.

***Abstract*—Recent studies have shown that the interaction between a finite momentum Cooper-pair boson and a fermion supports superconductivity in cuprates. Specifically, the occurrence of a superconducting energy gap in cuprates is defined by the collective excitation of boson-fermion pair condensates (BFPC) above the ground state. The ground state energy of BFPC in these materials show dependence on single-fermion spins. We study the specific heat of a BFPC in high temperature cuprates based on the ground state energy which represents the total internal energy of the system. The model results are in close proximity with the empirical findings. For instance, the specific heat of the model in Y123 is determined as $118.9\,\mathrm{Jmol^{-1}K^{-1}}$, which close to the measured value of $127.8\,\mathrm{Jmol^{-1}K^{-1}}$.**

Keywords—BFPC, ground state energy, Specific heat

I. Introduction

The specific heat of a material is derived from its total internal energy. Theoretically, the total internal energy is represented by the temperature-dependent ground state energy of a system. In high temperature superconductors, the specific heat in superconducting state of a material increases exponentially with temperature up to T_c [1]. Beyond T_c, the condensate system breaks down, paving way for another form of electronic interactions thereby causing a significant variation in the specific heat. The phenomenon of high temperature superconductivity is driven by the collective rather than single-particle behaviour of boson-fermion pairs [2][3]. Furthermore, superconductivity in cuprates show dependence on single fermion spin [3].

Experimentalists have relied on the specific heat jump between normal and superconducting states – Sommerfeld's coefficient – to determine specific heat values of high temperature superconductors. In experiments, specific heat has been determined from the Sommerfeld coefficient [4]5][6]. At $T = T_c$, two values for specific heat arise: the specific heat due to the superconducting state C_S and that due to the normal state C_n. The jump in specific heat has been expressed in terms of the applied external magnetic field using the *Rutgers' formula* given by equation (1) [7] (Kim *et al*., 2013).

$$C_S - C_n = \frac{T}{4\pi}\left(\frac{dH}{dT}\right)^2 \quad (1)$$

Where, H is the external applied field whose maximum is at the critical magnetic field, H_c. **Figure 1** shows an experimentally generated graph of $\frac{C_V}{T}$ as a function of temperature for varied hole-doping in thallium-based superconductor.

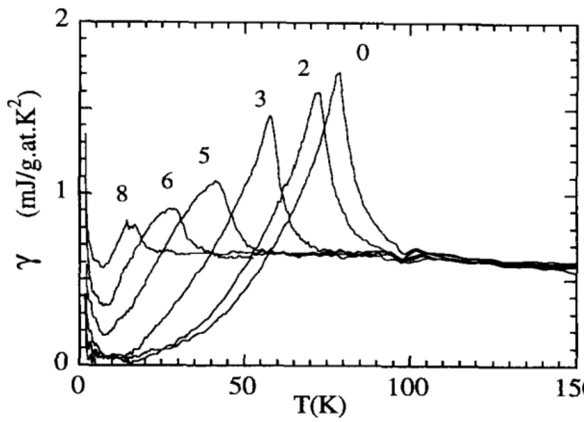

Figure 1: Sommerfeld's coefficient as a function of temperature for $Tl_2Ba_2CuO_{6+\delta}$ in the range $0 \leq \delta \leq 0.1$. The values indicated represent 100δ. Adapted from Loram *et al*. [4].

It is observed from **Figure 1** that an increase in the hole content (δ) causes a drop in the Sommerfeld's coefficient. At T_c, the change in $\frac{C_V}{T}$ occurs at constant temperature (thus, expressed as $\frac{\Delta C_v}{T}$). ΔC_v is about 1-2% of the phonon contribution to the total specific heat [4][5]. Elsewhere the contribution by ΔC_v to the specific heat has been estimated at 3.86% [6]. The quality of a sample influences the specific heat obtained from a sample. Poor quality samples produce lower values while good quality samples possess higher values of Sommerfeld's coefficient and hence specific heat [7][8].

On the other hand, theoretical physicists have derived specific heat from the internal energy of a system operating in a given material. Annavarapu [9] has noted that contributions from electrons and plasmons may open up a new insight into the thermodynamic properties of cuprates. Several models have been developed in an attempt to explain the high-energy pairing but they are not consistent with experimental findings. Recently, Mukubwa and Makokha [3] developed a boson-fermion model whose collective excitation energy is in tandem with the measured energy gaps in several cuprate materials. The model has also shown anisotropy due to Coulomb attraction.

II. Theoretical Formulations

The average temperature independent total ground state energy per spin in millielectron-volts is given by Mukubwa and Makokha [3] as

$$E_0^{av} = -0.83E_K^2 \quad (2)$$

Where q is the electron charge while $E_k = \frac{1}{q}(2k_BT_c)$ is the maximum single-particle ground state energy. The specific heat of a BFP condensate is expressed as

$$C_v = \left(\frac{\partial |E_0|}{\partial T}\right)_v = 4\left(\frac{T_c}{T^2}\right)(0.83E_K^2)\exp\left(-\frac{2T_c}{T}\right) \quad (3)$$

The molar specific heat in Joules of the system becomes

$$C_v = 3.33qN_A\left(\frac{T_c}{T^2}\right)(E_k^2 \times 10^{-3})\exp\left(-\frac{2T_c}{T}\right) \quad (4)$$

Here, N_A is the Avogadro's number.

III. Results and Discussions

After The graphs of specific heat of excited BFP pair condensates as a function of temperature in layered superconductors have been plotted in **Figure 5.2** using equation (4).

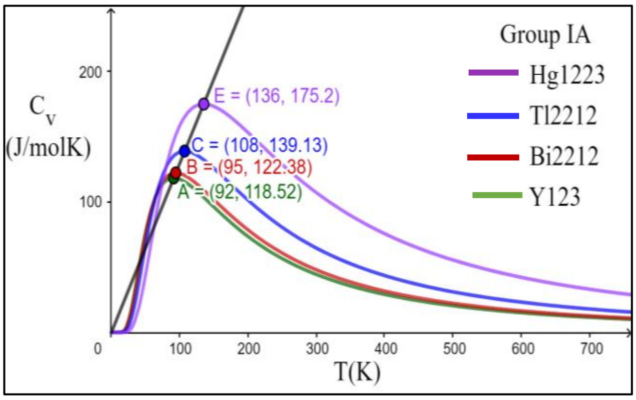

Figure 2: Specific heat of a BFP condensate in layered superconductors as a function of temperature

From the graph, the specific heat of a BFP condensate increases with temperature and reaches its peak at $T = T_C$. The value of the specific heat of the model in Y123 is $118.5\,\mathrm{Jmol^{-1}K^{-1}}$ while the experimental specific heat of Y123 is $127\,\mathrm{Jmol^{-1}K^{-1}}$ as determined by Loram *et al.* [4] and $125\,\mathrm{Jmol^{-1}K^{-1}}$ by Shaviv *et al.* [6]. Further comparisons between the model specific heats in layered superconductors vis a vis the experimental values for the materials is shown in **Table 1**.

Superconductor	$T_c(K)$	$C_{v(max)}$		**Ref.**
		Model	**Exp**	
$YBa_2Cu_3O_7$	92	118.5	127	[4]
$HgBa_2Ca_2Cu_3O_8$	136	175.2	~200	[10]
$Tl_2Ba_2CaCu_2O_8$	108	139.1	166	[11]

Table 1: *Comparison between the specific heat of the model and experimental specific heats of cuprate superconductors*

On the other hand, the specific heat of the model in Hg1223 has been determined as $175.2\,\mathrm{Jmol^{-1}K^{-1}}$ while Wesche [10] has reported the measured specific heat of Hg1223 as being slightly over $200\,\mathrm{Jmol^{-1}K^{-1}}$. The specific heat of $YBa_2Cu_3O_7$ based on a Bose-Fermi Hubbard model is found to be $450\,\mathrm{Jmol^{-1}K^{-1}}$ [1]. Generally, the specific heat of the BFPC model in various cuprates are in close proximity to the empirical results. The specific heat of a BFP condensate shows a linear dependence on the critical temperature (see **Figure 2)**) and follow the relation:

$$C_v = 1.29T_c \tag{5}$$

Conclusion

The interaction energy between bosons and fermions at ground state increases with temperature and hence a rise in the specific heat of the system. The specific heat of bosons-fermion pair condensate in cuprates is in close proximity with the empirical values and shows linear dependence on critical temperature of a material defined by equation (5).

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