(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

COURSE CODE: **MAP 322**

COURSE TITLE: GROUP THEORY II

DATE: 14/04/23 **TIME:** 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30MARKS)

a. Define the following
   i. The center of a Group (2marks)
   ii. Conjugacy class (2marks)

b. State the consequence of the class equation (2marks)

c. Work out the Conjucacy classes of the symmetric group S_3 (4 marks)

d. Work out the center of D_4 (2 marks)

e. Let G be a group of order p^n where p is prime. Show that G has a non-trivial center (5 marks)

f. Define the following
   i. P- subgroup (2 marks)
   ii. Normalizer (2 marks)

g. State the following theorems
   i. First Sylow theorem (2 marks)
   ii. Third Sylow theorem (2 marks)

h. Find the order of the alternating group A_5. Hence by Cauchy's theorem find the order of its subgroups (5 marks)

QUESTION TWO (20MARKS)

a. Define the following
   i. Maximal normal subgroup (2 marks)
   ii. Supersoluble (2 marks)
   iii. Chief series (2marks)
   iv. Soluble (2marks)

b. Show that all finite abelian groups are soluble (6 marks)

c. Show that every finite group has a composition series (6 marks)

QUESTION THREE (20MARKS)

a. Define the following
   i. Nilpotent group (2marks)
   ii. Nilpotency class (2marks)
   iii. Central series (2 marks)
   iv. Lower central series (2 marks)

b. Show that every nilpotent group is solvable (6 marks)

c. Show that a group G is nilpotent if and only if it has a central series (6marks)

QUESTION FOUR (20MARKS)

a. State the following theorems
   i. Fundamental theorem of finite abelian groups (3 marks)
   ii. Fundamental theorem of finitely generated abelian groups (3marks)

b. Use the fundamental theorem of finite abelian groups to classify all abelian groups of order 540. (7marks)

c. Show that a finite abelian group is a p-group if and only if its order is a power of p (7 marks)

QUESTION FIVE (20MARKS)

a. Define the following
   i. External direct product (2marks)
   ii. Internal direct product (2marks)

b. Show that if G is the internal direct product of H and K, then G is isomorphic to the external direct product $H \times K$ (11 marks)

c. Let G be a group with subgroups H and K. Suppose that $G = HK$ and $H \cap K = \{ 1_G \}$. Show that every element g of G can be written uniquely as hk for $h \in H$ and $k \in K$ (5 marks)