(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE: **MAP 223**

COURSE TITLE: **ALGEBRAIC STRUCTURES II**

DATE: 17/4/2023 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following terms
   i. Direct product of groups (2marks)
   ii. Symmetric group (1marks)
   iii. Ring (5marks)

b) Prove that for some $n \in \mathbb{N}$, $\sum_{k=1}^{n} k^2 = \frac{1}{6} n(n+1)(2n+1)$ (6marks)

c) Let G be the group $\{e, a, b, b^2, b^3 ab, ab^2, ab^3\}$ whose generators satisfy $a^2 = e, b^4 = e$, $ba = ab^3$. Construct the Cayley table of G and prove if it's abelian (12marks)

d) Let $H = \{x \in G: x = y^2 \text{ such that } y \in G\}$ Prove that H is a subgroup of G (4marks)

QUESTION TWO (20 MARKS)

a) Define the following terms
   i. Binary operation (2marks)
   ii. Residue classes mod n (3marks)
   iii. Group Isomorphism (3marks)

b) Discuss if the below are binary operations. If not give an explanation
   i. Subtraction on the set $\{n \in \mathbb{Z}: \geq 0\}$ (3marks)
   ii. $a * b$ is a root of the equation $x^2 - a^2b^2 = 0$ on the set $\mathbb{R}$ (3marks)

c) List the elements of $\mathbb{Z}_2 \times \mathbb{Z}_4$ and write its operation table (6marks)

QUESTION THREE (20 MARKS)

a) Define the following terms
   i. Group (4marks)
   ii. Generators of a group (2marks)
   iii. Cyclic group (2marks)
   iv. Lagrange's theorem (2marks)

b) List all the elements of S_3 in stack and cycle notation, state the order of each element and verify the Lagrange's theorem for S_3 (10marks)

QUESTION FOUR (20 MARKS)

a) Define the following terms
   i. Multiplicative inverse (2marks)
   ii. Subgroup (3marks)

b) State residue classes of integers mod 3 (3marks)

c) Solve simultaneously $x^2a = bxc^{-1} \text{ and } acx = xac$ (3marks)

d) Express the elements of the Dihedral group D_4 in cycle notation (4marks)

e) Find cyclic subgroups of S_4 of orders 2,3 and 4 (5marks)

QUESTION FIVE (20 MARKS)

a) Prove that the inverse in any group G is unique (3marks)

b) Let G_1 and G_2 be groups and ϕ: $G_1 \times G_2 \rightarrow G_1 \times G_2$. Prove that ϕ is an isomorphism (6marks)

c) Prove by induction if $ab = ba$, then $(ab)^n = a^n b^n$ (5marks)

d) State the Division algorithm (2marks)

e) Let G be a cyclic group of order 10. How many of its elements generate G? (4marks)