(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FORTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE: **MAP 413**

COURSE TITLE: **FUNCTIONAL ANALYSIS**

DATE: 25/4/2023 **TIME: 9 AM – 11 AM**

INSTRUCTIONS TO CANDIDATES

Answer question ONE and any other two questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms

   i. Metric space

   ii. Normed space

b) Let X be a set of all real valued functions, x, y, z of an independent variable t that are defined and continuous on an interval $\tau = [a, b]$. Denote the set $C[a, b]$ as $C[a, b] = \{x: x(t)\}$ is defined and contionous on $[a, b]$ and let a metric on $C[a, b]$ be given by $d(x, y) = \max_{t\in\tau} |x(t) - y(t)|$. Show that this set together with the metric d is a metric space

QUESTION TWO (20 MARKS)

a) Define the following terms

   i. Bounded set

   ii. Bounded sequence

b) Given $X = (x, d)$ is a metric space show that

   i. A convergent sequence X is bounded and its limit is unique

   ii. If $x_n \to x$ and $y_n \to y$ in X, then $d(x_n, y_n) \to d(x, y)$

c) Show that every convergent sequence in a metric space is a Cauchy sequence

QUESTION THREE (20 MARKS)

a) Show that the space $\mathcal{L}^\infty$ is complete

b) Show that the space $\mathcal{L}^p$ $1 \leq p \leq \infty$ is complete

c) Give an example of an incomplete metric space

QUESTION FOUR (20 MARKS)

a) Define the following terms

   i. Complete space

   ii. Banach space

b) Show that the Euclidean space $\mathbb{R}^n$ is a normed space

c) Show that the space C[a,b] with norm given by $||x|| = max_{t\ni\tau}|x(t)|$ $where$ $\tau = [a, b]$ is a Banach space

QUESTION FIVE (20 MARKS)

a) Show that every finite dimensional subspace Y of a normed space X is complete

b) Define the terms

   i. Equivalent norms

   ii. Compactness

c) State Riesz's Lemma