(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

(MATHEMATICS)

COURSE CODE: **MAA 121**

COURSE TITLE: **FOUNDATION MATHEMATICS II**

DATE: 19/04/23 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

(a) If $\vec{P}$ and $\vec{Q}$ are vectors, prove that $\vec{P} \times \vec{Q} = -(\vec{Q} \times \vec{P})$ (5 mks)

(b) Determine the values of x, y and z by reducing the given system of linear equations to echelon form, (5 mks)

$$-2x + z - y = -6$$
$$3x + 2y - z = 4$$

(c) Given $\boldsymbol{a} = 4i - 9j + 2k$ and $\boldsymbol{b} = 2j + 6k$ find the projection of $\boldsymbol{b}$ on $\boldsymbol{a}$ (6 mks)

(d) Given that I is an identity matrix find A if $(6I + A^T)^{-1} = \begin{bmatrix} -5 & 10 \\ 3 & -2 \end{bmatrix}$ (5 mks)

(e) Find the inverse of the matrix using matrix inversion algorithm

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$ (9 mks)

QUESTION TWO (20 MARKS)

(a) Show that $\|\boldsymbol{a} \times \boldsymbol{b}\| = \|\boldsymbol{a}\|\|\boldsymbol{b}\| sin\theta$ (5 mks)

(b) Find the angle between the vectors $3i - 2j - k$ and $-4i + j - 2k$ (4 mks)

(c) Given that $M = \begin{bmatrix} 6 & -3 & 5 \\ 2 & 3 & -6 \\ 1 & 3 & 7 \end{bmatrix}$ and $N = \begin{bmatrix} -4 & -10 & 7 \\ 1 & 2 & 3 \\ 2 & -1 & 0 \end{bmatrix}$

Prove that $detMdetN = \det(MN)$ (5 mks)

(d) Solve the system by Gauss-Jordan elimination (6 mks)

$$4x + y + z = 4$$
$$x + 4y - 2z = 4$$
$$3x - 4z + 2y = 6$$

QUESTION THREE (20 MARKS)

(a) Define the following matrices giving an example in each case

(i) Square matrix (2 mks)

(ii) Diagonal matrix (2 mks)

(b) If $detA = 15$ and $\det B = -3.2$ calculate $det(A^2B^{-1}B^3AB^T)$, given that matrices A and B are square matrices (3 mks)

(c) Use Cramer's rule to find $x_1, x_2,$ and $x_3,$ (8 mks)

$$x_1 + x_2 + x_3 = 1$$

$$4x_1 + 3x_2 - x_3 = 6$$

$$3x_1 + 5x_2 + 3x_3 = 4$$

(d) Compute the determinant of $\begin{bmatrix} 4 & 1 & -2 & 2 \\ 1 & 2 & 0 & 1 \\ -2 & 0 & 3 & -2 \\ 2 & 1 & -2 & -1 \end{bmatrix}$ (5 mks)

QUESTION FOUR (20 MARKS)

(a) (i) Find λ so that $4\lambda i - \lambda j + 10k$ and $\lambda i - j - 2k$ are perpendicular. (3 mks)

(ii) A plane is defined by 3 points P(1,0,-1), Q(2,1,1) and R(1,-1,1), find a vector perpendicular to the plane. (5 mks)

(b) If $A = \begin{bmatrix} a+x & 2x & p \\ b+y & 2y & q \\ c+z & 2z & r \end{bmatrix}$ Evaluate $detA$ given that

$$det\begin{bmatrix} a & p & x \\ b & q & y \\ c & r & z \end{bmatrix} = 10$$ (4 mks)

(c) Compute the rank of

$$\begin{bmatrix} -1 & 3 & 0 & 3 & 1 & 3 \\ 1 & -3 & 1 & -1 & 0 & -1 \\ -1 & 3 & 1 & 5 & 1 & 6 \\ 2 & -6 & 3 & 0 & -1 & -1 \end{bmatrix}$$ (8 mks)

QUESTION FIVE (20 MARKS)

(a) Find the solution of the following system of linear equations using Gaussian elimination with backward substitution (10mks)

$$x_1 + x_3 - 2x_4 = -3$$
$$2x_1 + x_2 - x_3 = 2$$
$$-4x_1 + x_2 - 7x_3 - x_4 = -19$$
$$4x_1 + 2x_2 + x_3 - 3x_4 = -2$$

(b) Given the matrix

$A = \begin{bmatrix} -1 & 6 & 4 \\ -3 & 0 & -5 \\ 4 & 1 & 6 \end{bmatrix}$, Compute $A(adjA)$ (10 mks)