



(Knowledge for Development)

# KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

**2022/2023 ACADEMIC YEAR** 

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE:

**MAA 224** 

COURSE TITLE:

ORDINARY DIFFERENTIAL EQUATIONS I

**DATE**: 13/4/2023

TIME: 9:00 AM - 11:00 AM

# **INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

### **QUESTION ONE COMPULSORY (30 MARKS)**

- (a) Define the following terns
  - i. Differential equation
  - ii. Initial Value Problem
  - iii. Order of differential equation (3 marks)
- (b) Determine the order and linearity of the following ordinary differential equation

$$\left(\frac{d^5y}{dx^5}\right)^{\frac{7}{2}} + 4\left(\frac{dy}{dx}\right)^6 = x^{10}y$$
 (2 marks)

- (c) Using an appropriate method, solve  $\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$  (4 marks)
- (d) Use operator method to solve  $\frac{d^3y}{dx^3} 2\frac{d^2y}{dx^2} 5\frac{dy}{dx} + 6y = e^{3x}$  (6 marks)
- (e) Show that  $\frac{dy}{dx} = \frac{x^2}{y + x^3 y}$  is separable and find its particular solution under the condition y(0) = -2 (4 marks)
- (f) Verify that the functions x and  $xe^x$  are linearly independent solutions of

$$x^{2} \frac{d^{2}y}{dx^{2}} - x(x+2)\frac{dy}{dx} + (x+2)y = 0$$
 (6 marks)

(g) Test for exactness and solve the following ordinary differential equation  $(\cos x - x \sin x + y^2)dx + 2xy dy = 0$  (5 marks)

### **QUESTION TWO (20 MARKS)**

(a) Solve 
$$\frac{dy}{dx} = \frac{xy^2 + x}{x^2y + y}$$
 (4 marks)

(b) Solve 
$$(3y - 7x + 7)dx - (7y - 3x + 3)dy = 0$$
 (7 marks)

(c) A loaf of bread is removed from an oven at a temperature of 300  $^{0}c$  and placed in a room whose temperature is 70  $^{0}c$ . 3 minutes later, the temperature of the bread is  $200 \, ^{0}c$ .

i. Find its temperature function at any time t. (7 marks)

ii. How long will it take to cool to  $150^{\circ}c$ . (2 marks)

## **QUESTION THREE (20 MARKS)**

- (a) Write the standard form of a linear first order ordinary differential equation giving an example. (2 marks)
- (b) Use operator method to solve  $2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = x$  (7 marks)
- (c) Solve the non-linear equation  $x^2 \frac{dy}{dx} 2xy = 3y^4$  (6 marks)
- (d) Solve the Cauchy-Euler's equation

$$x^{3} \frac{d^{2}y}{dx^{2}} + 5x^{2} \frac{d^{2}y}{dx^{2}} + 7x \frac{dy}{dx} + 8y = 0$$
 (5 marks)

### **QUESTION FOUR (20 MARKS)**

(a) Use the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x^2 + 1}$$
 (8 marks)

(b) Test for exactness then solve the following differential equations

i. 
$$(y^2 - x)dx + 2ydy = 0$$
 (7 marks)

ii. 
$$(\cos(x+y^2)+3y) dx + (2y\cos(x+y^2)+3x))dy = 0$$
 (5 marks)

## **QUESTION FIVE (20 MARKS)**

(a) Use the method of undetermined coefficients to solve

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = x^2 + x$$
 (8 marks)

(b) Solve the following differential equations using appropriate method

i. 
$$\frac{dy}{dx} = \frac{xy + y^2}{x^2 - xy}$$
 (5 marks)

ii. 
$$(x + 2y - 1)dx + (3x + 6y)dy = 0$$
 (7 marks)