(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

COURSE CODE: **MAP 412**

COURSE TITLE: **MEASURE THEORY**

DATE: 28/4/2023 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer question ONE and any other two questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms

i. Ring

ii. Algebra

iii. $\sigma - ring$

iv. $\sigma - algebra$

b) Show that if $f: X \to Y$ and F is a $\sigma - ring$ of subsets of Y , then the class of all sets of the form $f^{-1}(B)$ where B is a $\sigma - ring$ of subsets of X

c) State the lemma on Monotone classes (LMC)

QUESTION TWO (20 MARKS)

a) Define a measure

b) Show that if R is a ring and μ is an extended real valued set function on R which is positive, countably additive and satisfies the condition $\mu(\emptyset) = 0$, then μ is a measure

c) Define the terms

i. Countably additive

ii. Contraction

QUESTION THREE (20 MARKS)

a) Show that if μ is measurable on a ring R, and define an extended real valued set function μ^* on H(B) by the formula $\mu^* = GLB\{\sum_1^\infty \mu(E_n): A \subset U_1^\infty E_{n,} \quad E_n \in \mathbb{R}, (n = 1,2, \ldots)\}$ then

i. μ^* is positive

ii. $\mu^*(\emptyset) = 0$

iii. μ^* is monotone

iv. μ^* is countable sub additive

v. μ^* extends μ

b) Define the following terms

i. Outer measure

ii. v-measure

c) Show that if v is an outer measure, then the class M of v −measurable sets is a ring

QUESTION FOUR (20 MARKS)

a) State the Unique Extension Theorem

b) Show that if $\alpha_i \uparrow \alpha$ and $\beta_i \uparrow \beta$ then $\alpha_i\beta_i \uparrow \alpha\beta$

c) Define a simple function

QUESTION FIVE (20 MARKS)

a) Explain the following terms

i. $f = g$ a.e

ii. $f \leq g$ a.e

iii. $f = constant$ a.e

iv. f is essentially bounded

b) Show that if f_n is a sequence of integrable functions such that $f_n \geq 0$ $a.e$ and $\liminf \int f_n \, du < \infty$ then there exists an integrable function f such that $f = \liminf f_n$ a.e and one has $\int f \, d\mu \leq \liminf \int f_n \, d\mu$