KIBABII UNIVERSITY

MAIN UNIVERSITY EXAMINATIONS

ACADEMIC YEAR 2022/2023

THIRD YEAR SECOND SEMESTER EXAMINATIONS

BACHELOR OF SCIENCE (PHYSICS)

COURSE CODE: SPC 323

COURSE TITLE: MATHEMATICAL PHYSICS II

DATE: 21/04/2023 **TIME: 2:00-4:00PM**

INSTRUCTIONS TO CANDIDATES

Answer question ONE and any TWO of the remaining.

Time: 2 hours

KIBU observes ZERO tolerance to examination cheating

QUESTION ONE (30 MARKS)

a) Show that u(x,t) =$e^y\sin(x)$ is a solution to Laplace's equation (3 marks)

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

b) Using Cauchy's integral formula, evaluate I= $\oint_C \frac{dz}{z(z+2))}$ with C on a unit circle

(3 marks)

c) Using the inner product of a tensor and applying the contraction principle, obtain the length L of a tensor A^i where A^i is a vector. (3 marks)

d) Compute Laurent expansion about Z_o=0 at $f(z)=\frac{1}{z(z-1)}$ (3 marks)

e) Use Laplace transforms to evaluate, $f(t)=\cosh(kt)=\frac{1}{2}\left(e^{kt}+e^{-kt}\right)$ (3 marks)

f) Find the Fourier transform of $f(t)=e^{-\alpha|1|}$ where α>0, (3 marks)

g) Use the Fourier integral to prove that (3 marks)

$$\int_0^\infty \frac{\cos axdx}{1+a^2}=\frac{\Pi}{2}e^{-x}$$

h) Prove that the Legendre polynomials satisfy the following: (3 marks)

$$(2n+1)p_n(x) = p^{I}_{n+1}(x) - p^{I}_{n-1}(x)$$

i) Determine the residues of the following functions at the poles z=1 and z=-2

(3 marks)

$$\frac{1}{(z-1)(z+2)^2}$$

j) Find the Laurent series about the singularity for the function: (3 marks)

$$\frac{e^z}{(z-2)^2}$$

QUESTION TWO (20 MARKS)

a) Consider the chain decay in radioactivity $A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C$ where λ_A and λ_B are the disintegration constants. The equations of for the radioactive decays are:

$$\frac{dN_A(t)}{dt} = -\lambda_A N_A(t), \text{ and } \frac{dN_B(t)}{dt} = -\lambda_2 N_B(t) + \lambda_A N_A(t)$$

Where $N_A(t)$ and $N_B(t)$ are the number of atoms of A and B at time t, with initial conditions $N_A(A) = N_A^0;\ N_B(0) = 0$. Apply Laplace transform to obtain $N_A(t)$ and $N_B(t)$, the number of atoms of A and B as a function of time t, in terms of N_A^0 , λ_A and λ_B (10 marks)

b) Consider the radioactive decay:

$$A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C \text{ (stable)}$$

The equations for the chains are:

$$\frac{dN_A}{dt} = -\lambda_A N_A \quad (1)$$

$$\frac{dN_B}{dt} = -\lambda_B N_B + \lambda_A N_A \quad (2)$$

$$\frac{dN_C}{dt} = +\lambda_B N_B \quad (3)$$

with initial conditions $N_A(0) = N_A^0;\ N_B(0) = 0\ ;\ N_C(0) = 0$, where various symbols have usually meaning. Apply Laplace transforms to find the growth of C. (10 marks)

QUESTION THREE (20 MARKS)

Find the solution of the one dimensional wave equation describing the motion of a string and given as $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ subject to boundary conditions $u(0,t) = 0$ and $u(l,t) = 0$ (20 marks)

QUESTION FOUR (20 MARKS)

a) Show that the Legendre polynomials have the property (10 marks)

$$\int_{-l}^{l} P_n(x) P_m(x)\, dx = \frac{2}{2n+1}, \text{ if } m = n$$
$$= 0, \text{ if } m \neq n$$

b) Show that for large n and small θ, $p_n(\cos\theta) \approx J_0(n\theta)$ (10 marks)

30

QUESTION FIVE (20 MARKS)

a) What are contravarient, co-varient and mixed tensors? Show that velocity and acceleration are contravarient and the gradient of a field is a covariant tensor. (10 marks)

b) Give a short account of metric tensors and its applications (10 marks)