



**KIBABII UNIVERSITY**

**MAIN UNIVERSITY EXAMINATIONS**

**ACADEMIC YEAR 2022/2023**

**THIRD YEAR SECOND SEMESTER EXAMINATIONS**

**BACHELOR OF SCIENCE (PHYSICS)**

**COURSE CODE: SPC 323**

**COURSE TITLE: MATHEMATICAL PHYSICS II**

**DATE: 21/04/2023**

**TIME: 2:00-4:00PM**

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**INSTRUCTIONS TO CANDIDATES**

**Answer question ONE and any TWO of the remaining.**

**Time: 2 hours**

**KIBU observes ZERO tolerance to examination cheating**

**QUESTION ONE (30 MARKS)**

- a) Show that  $u(x,y) = e^y \sin(x)$  is a solution to Laplace's equation (3 marks)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

- b) Using Cauchy's integral formula, evaluate  $I = \oint_C \frac{dz}{z(z+2)}$  with C on a unit circle (3 marks)

- c) Using the inner product of a tensor and applying the contraction principle, obtain the length L of a tensor  $A^i$  where  $A^i$  is a vector. (3 marks)

- d) Compute Laurent expansion about  $Z_0=0$  at  $f(z) = \frac{1}{z(z-1)}$  (3 marks)

- e) Use Laplace transforms to evaluate,  $f(t) = \cosh(kt) = \frac{1}{2}(e^{kt} + e^{-kt})$  (3 marks)

- f) Find the Fourier transform of  $f(t) = e^{-\alpha|t|}$  where  $\alpha > 0$ , (3 marks)

- g) Use the Fourier integral to prove that (3 marks)

$$\int_0^{\infty} \frac{\cos ax dx}{1+a^2} = \frac{\pi}{2} e^{-x}$$

- h) Prove that the Legendre polynomials satisfy the following: (3 marks)

$$(2n+1)p_n(x) = p_{n+1}'(x) - p_{n-1}'(x)$$

- i) Determine the residues of the following functions at the poles  $z=1$  and  $z=-2$  (3 marks)

$$\frac{1}{(z-1)(z+2)^2}$$

- j) Find the Laurent series about the singularity for the function: (3 marks)

$$\frac{e^z}{(z-2)^2}$$

**QUESTION TWO (20 MARKS)**

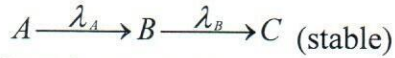
- a) Consider the chain decay in radioactivity  $A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C$  where  $\lambda_A$  and  $\lambda_B$  are the disintegration constants. The equations of for the radioactive decays are:

$$\frac{dN_A(t)}{dt} = -\lambda_A N_A(t), \text{ and } \frac{dN_B(t)}{dt} = -\lambda_B N_B(t) + \lambda_A N_A(t)$$

Where  $N_A(t)$  and  $N_B(t)$  are the number of atoms of A and B at time t, with initial conditions  $N_A(0) = N_A^0$ ;  $N_B(0) = 0$ . Apply Laplace transform to obtain  $N_A(t)$  and

$N_B(t)$ , the number of atoms of A and B as a function of time t, in terms of  $N_A^0$ ,  $\lambda_A$  and  $\lambda_B$  (10 marks)

- b) Consider the radioactive decay:



The equations for the chains are:

$$\frac{dN_A}{dt} = -\lambda_A N_A \tag{1}$$

$$\frac{dN_B}{dt} = -\lambda_B N_B + \lambda_A N_A \tag{2}$$

$$\frac{dN_C}{dt} = +\lambda_B N_B \tag{3}$$

with initial conditions  $N_A(0) = N_A^0$ ;  $N_B(0) = 0$ ;  $N_C(0) = 0$ , where various symbols have usually meaning. Apply Laplace transforms to find the growth of C. (10 marks)

**QUESTION THREE (20 MARKS)**

Find the solution of the one dimensional wave equation describing the motion of a string and

given as  $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$  subject to boundary conditions  $u(0,t) = 0$  and  $u(l,t) = 0$  (20 marks)

**QUESTION FOUR (20 MARKS)**

- a) Show that the Legendre polynomials have the property (10 marks)

$$\int_{-1}^1 P_n(x) P_m(x) dx = \frac{2}{2n+1}, \text{ if } m = n$$
$$= 0, \text{ if } m \neq n$$

- b) Show that for large n and small  $\theta$ ,  $P_n(\cos\theta) \approx J_0(n\theta)$  (10 marks)

**QUESTION FIVE (20 MARKS)**

- a) What are contravariant, co-variant and mixed tensors? Show that velocity and acceleration are contravariant and the gradient of a field is a covariant tensor. (10 marks)
- b) Give a short account of metric tensors and its applications (10 marks)