(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION

COURSE CODE: **MAA 225**

COURSE TITLE: COMPLEX ANALYSIS I

DATE: 12/04/2023   **TIME:** 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **2** Printed Pages. Please Turn Over.

Question One (30 marks)

(a) Define the following terms giving examples where necessary .

(i) Neighborhood of z_o (2 mks)

(ii) Deleted Neighborhood of z_o (2 mks)

(iii) Interior point   (2 mks)

(iv) Open set   (2 mks)

(v) Boundary point   (2 mks)

(vi) Exterior point   (2 mks)

(b) If $(x+y)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = (x^2 + y^2)$ (8 mks)

(c) Express $\frac{1}{1-\cos\theta+2i\sin\theta}$ in the form a + ib.   (4 mks)

(d) Show that the polynomial $x^{4p} + x^{4q+1} + x^{4r+2} + x^{4s+3}$ is divisible by $x^3 + x^2 + x + 1$. where $p, q, r, s \in \mathbb{N}$ (6 mks)

Question Two (20 marks)

(a) Evaluate $(-\sqrt{-1})^{4n+3}$ for $n \in \mathbb{N}$ (2 marks)

(b) Find the value of $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ for $n \in \mathbb{N}$ (3 marks)

(c) Express $(-\sqrt{-3} - i)$ in polar form   (5 mks)

(d) Find the fourth roots of $z = 1 + i$ (10 mks)

Question Three (20 marks)

(a) Prove that $lim_{z \longrightarrow 1+i} (2 + i)z = 1 + 3i$ (15 mks)

(b) Show that the function $f(z) = x^2 - y^2 + x + i(2xy + y)$ is analytic   (5 mks)

Question Four (20 marks)

Evaluate the following

$\int_{-\infty}^{\infty} \frac{1}{(x^2+1)(x^2+9)} dx$ (20 mks)

Question Five (20 marks)

(a) Verify that the function $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic in the entire complex plane   (5 mks)

(b) Find the harmonic conjugate function of u in (a) above   (5 mks)

(c) Evaluate

$\int_0^{2\pi} \frac{1}{(2+\cos\theta)^2} d\theta$ (10 mks)