(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 252**

COURSE TITLE: **ENGINEERING MATHEMATICS II**

DATE: 18/4/2023 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

(a) Differentiate between thermodynamics and Magneto hydrodynamics in relation to fluid motion. (2marks)

i) In a adiabatic expansion of a gas, $C_v \frac{dP}{P} + C_P \frac{dV}{V} = 0$, where C_p and C_v are constants. Given $n = \frac{Cp}{C_v}$ show that pv^n= Constant using method of separation of variables (3 marks)

(b) Determine a pair of phasors that can be used to represent the following voltage

$\mathbf{V} = 8cos2t$ (2 marks)

(c) For **R-C** determine the resistance and series inductance for the following assuming the frequency of 50**Hz** (4marks)

i) $(4 + i7)\mathbf{\Omega}$

ii) $-i20\mathbf{\Omega}$

(d) A production company for renewable energy has 35 similar solar powered machines. The number of breakdowns on each machines average 0.06 $per\ week$. Determine probabilities of having less than 3 machines breaking down in any week (3marks)

(e) Consider a function $f(z) = z^2$ write in terms of r and θ in modulus Argument form (3 marks)

(f) Convert $f_r(x) = 5cosx + 12sinx$ into complex form $(f_c(x))$ and state the Fourier coefficients. (4 marks)

(g) A hydroelectric power was laid in a gradient defined by an engineer a $f(x, y, z) = x^2z^2\sin(4y)$. Find the gradient of the function using the definition of gradient. (3marks)

(h)i) Use the line integral to compute work done by a force $\vec{\mathbf{F}} = (2y + 3)\tilde{\imath} + (xz)\tilde{\jmath} + (yz - x)\tilde{k}$ .When it moves a particle from the point (0, 0, 0) to the point (2, 1, 1) along the curve $x = 2t^2,\ y = t.$ (4marks)

(ii)State Greens theorem. (2marks)

QUESTION TWO (20 MARKS)

(a) Use the Laplace transform method to solve the differential equation (10marks)

$2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 3y = 0$ .Given that $x = 0,\ y = 4$ and $\frac{dy}{dx} = 9$.

(b) The current flowing in an electrical circuit is given by the differential equation $\mathbf{R}i + \mathbf{L}\left\{\frac{di}{dt}\right\} = \mathbf{E}$, Where E, L and R are constants. Use the above method to Solve the equation for current i given that $t = 0$ and $i = 0$.
(10marks)

QUESTION THREE (20 MARKS)

(a) State two assumptions that describe the flow of heat in thermally conducting regions. (2marks)

(b) Find the solution of the initial boundary value problem for the heat equation $u_{xx} = h^2 u_t$ Satisfying the following initial boundary conditions

$\begin{cases} u_x(0,t) = 0, \\ u_x(l,t) = 0 \end{cases}$ $0 \le t < \infty$ and $u(x,0) = \sin\frac{\pi x}{l}$, $0 \le x \le \pi$ (10marks)

(c) Derive the stokes theorem in relation to vectors (8marks)

QUESTION FOUR (20 MARKS)

(a) Given $\vec{\mathbf{F}} = x^2 z\tilde{\imath} - 2y^3 z^2 \tilde{\jmath} + xy^2 z\tilde{k}$, Find $\nabla \cdot \vec{\mathbf{F}}$ at $(1,-1,1)$ (6marks)

(b) Find $Curl\ \vec{\mathbf{F}}$ (6marks)

(c) Consider a closed surface in space x, y, z enclosing a volume V as $\mathbf{A} = 45x^2y$ bounded by the planes $4x + 2y + z = 8$, $x = 0$, $y = 0$, $z = 0$. Evaluate
(8marks)

QUESTION FIVE (20 MARKS)

(a) Consider $f(z) = z^2$ evaluate and show that the function $f(z) = z^2$ is differentiable at every point in region $\mathbb{R}$. (8marks)

(b) Find the Laurent series of the function $f(z) = \frac{1}{(z-1)(z-2)}$ at $1 < |z| < 2$.
(8marks)

(c) If the above function is analytic on a simple closed contour $\boldsymbol{C}$ except at $\boldsymbol{Z_0}$.
Find the residues of the above function (4marks)