(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

COURSE CODE: **MAP 411**

COURSE TITLE: **TOPOLOGY**

DATE: 21/4/2023 **TIME**: 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a. Define the following

i. Topology (3 marks)

ii. Discrete topology (2marks)

iii. Cofinite topology (2marks)

iv. Finer topology (2marks)

b. Let X = {a, b} be a 2-element set. Write down four different topologies on X. (8 marks)

c. Let X= {a, b, c}. Give reasons why the following collections are not topologies on the set X

i. { {a}, {c}, {a ,c}, {a, b} } (1 mark)

ii. { ∅, {a}, {b}, X } (1 mark)

iii. { ∅, {a, b}, {a, c}, X } (1 mark)

d. Prove that Cofinite topology is a topology on the set X. (8 marks)

e. Show that the trivial topology is coarser than any other topology and the discrete topology is finer than any other topology. (2 marks)

QUESTION TWO (20 MARKS)

a. Define the following

i. Basis (3 marks)

ii. Topology generated by a basis (2marks)

b. Show that the collection T generated by a basis B is a topology on X (10 marks)

c. Let B be a Subbasis for X. Show that the associated collection B is a basis for a topology (5 marks)

QUESTION THREE (20 MARKS)

a. Define the term "Product topology" (2marks)

b. Show that the collection B= { U × V| U is open in X and V is open in Y} is a basis for a topology on X × Y (8 marks)

c. Let X have the topology generated by a basis B and Y topology generated by a basis C. Show that the collection D = { B ×C} is a basis for product topology on X × Y. (10 marks)

QUESTION FOUR (20 MARKS)

a. Define the following

i. Subspace topology (2marks)

ii. Open subspace (2marks)

b. Show that a subspace topology T_A is a topology on A (8 marks)

c. If B is a basis for a topology T on X and A⊂X, show that the collection BA = {B ∩A} is a basis for the subspace topology T_A on A (8 marks)

QUESTION FIVE (20 MARKS)

a. Define the following

i. Closed subset (2 marks)

ii. Closed subspace (2 marks)

b. Let X be a topological space. Show that

i. ∅ and X are closed subsets of X (2vmarks)

ii. The intersection of any closed subsets of X is closed (4 marks)

iii. The union of any finite collection of closed subsets of X is closed (4 marks)

c. Let A be a subspace of X. Show that a subset K ⊂ A is closed in A if and only if there exists a closed subset L ⊂ A with K= A∩L (6 marks)