

180



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
FIRST YEAR SECOND SEMESTER
MAIN EXAMINATION

FOR THE DEGREES OF BACHELOR OF EDUCATION ARTS

COURSE CODE: STA 112/STA 142

COURSE TITLE: INTRODUCTION TO PROBABILITY

DATE: 28/4/2023

TIME: 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 6 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- a) Consider the sample space $S = \{\text{copper, sodium, nitrogen, potassium, uranium, oxygen, zinc}\}$ and the events
 $A = \{\text{copper, sodium, zinc}\}$,
 $B = \{\text{sodium, nitrogen, potassium}\}$,
 $C = \{\text{oxygen}\}$.

List the elements of the sets corresponding to the following events: (4 mks)

- (i) $A \cap B \cap C$;
(ii) $(A^c \cup B^c) \cap (A^c \cap C)$

- b) Briefly discuss 4 properties of mathematical expectation. [4 marks]
- c) Let X be a random variable and $Y = 2X + 1$. What is the variance of Y if the variance of X is 5? [4 marks]
- d) If the probability of a defective bolt is 0.1, find
(i) The mean,
(ii) The standard deviation, for the number of defective bolts in a total of 400 bolts. (4mks)
- e) The mean weight of 500 male students at a certain college is 151 lb and the standard deviation is 15 lb. Assuming that the weights are normally distributed, find how many students weigh (4 mks)
(i) Between 120 and 155 lb,
(ii) More than 185 lb.
- f) If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001, determine the probability that out of 2000 individuals,
(i) Exactly 3,
(ii) More than 2, individuals will suffer a bad reaction. Assume X is Poisson distributed
- g) The probability distribution function of a discrete random variable X is

$$f(x) = \begin{cases} 2k, & x = 1 \\ 3k, & x = 3 \\ 4k, & x = 5 \\ 0, & \text{otherwise} \end{cases}$$

Where k is some constant.

Find,

- i. k [3 marks]
ii. $P(X > 2)$ [3 marks]

QUESTION TWO (20 MARKS)

(a) The probability distribution function of a continuous random variable X is

$$f(x) = \begin{cases} a + bx^2, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Where a and b are some constants, Find

[10 Marks]

- i. a and b if $E(X) = \frac{3}{5}$
- ii. $\text{Var}(x)$

(b) Verify the central limit theorem for a random variable X that is binomially distributed, and thereby establish the validity of the normal approximation to the binomial distribution. (10 mks)

QUESTION THREE (20 MARKS)

a) The number of miles an automobile tire lasts before it reaches a critical point in tread wear can be represented by a pdf

$$f(x) = \begin{cases} \frac{1}{30} e^{-\frac{x}{30}}, & \text{for } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the expected number of miles (in thousands) a tyre would last until it reaches the critical tread wear point.

[10 Marks]

b) Prove the (weak) law of large numbers for Bernoulli trials by Chebyshev's inequality (10mks)

QUESTION FOUR (20 MARKS)

(a) Box I contains 3 red and 2 blue marbles while Box II contains 2 red and 8 blue marbles.

A fair coin is tossed. If the coin turns up heads, a marble is chosen from Box I ; if it turns up tails, a marble is chosen from Box II .

i. Find the probability that a red marble is chosen. (5 mks)

ii. Suppose that the one who tosses the coin does not reveal whether it has turned up heads or tails (so that the box from which a marble was chosen is not revealed) but does reveal that a red marble was chosen. What is the probability that Box I was chosen (i.e., the coin turned up heads)? (5 mks)

(b) If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random,

(i) 1 bolts will be defective. (3 mks)

(ii) 0 bolts will be defective. (3 mks)

(ii) less than 2, bolts will be defective. (4 mks)

QUESTION FIVE (20 MARKS)

- a) Ten percent of the tools produced in a certain manufacturing process turn out to be defective. Find the probability that in a sample of 10 tools chosen at random, exactly 2 will be defective, by using
- (i) the binomial distribution, (5mks)
 - (ii) the Poisson approximation to the binomial distribution. (5mks)
- b) Establish the validity of the Poisson approximation to the binomial distribution. (10mks)

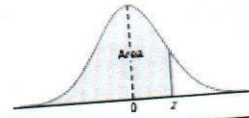


Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

