



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
SECOND YEAR SECOND SEMESTER
MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN
MATHEMATICS

COURSE CODE: MAA 223

COURSE TITLE: CLASSICAL MECHANICS

DATE: 12/04/23

TIME: 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- a) State three important properties of a central force field (3 marks)
- b) An artificial satellite is rotating about the earth at a height h of 200km above the surface of the earth where acceleration due to the gravity $g = 9.8\text{ms}^{-2}$. The radius of the earth r is 6400km . Calculate the speed of the satellite. (6 marks)
- c) A trolley M of mass 80g travelling at 3ms^{-1} collides with another trolley N of twice its mass moving in the opposite direction with a velocity of 0.5ms^{-1} . If the trolleys stuck together on collision, calculate the common velocity with which they move. (7 marks)
- d) Due to a force field a mass of 5units moves along a space curve whose position vector is $\underline{r} = (2t^3 + t)\underline{i} + (3t^4 - t^2 + 8)\underline{j} - 12t^2 \underline{k}$. Find
- (i) Velocity (3 marks)
- (ii) Momentum (3 marks)
- e) A constant force \vec{F} acting on a particle of mass m changes the velocity from v_1 to v_2 in time τ . Prove that $\vec{F} = \frac{m(v_2 - v_1)}{\tau}$ (8 marks)

QUESTION TWO (20 MARKS)

- a) Consider two blocks A and B of mass m_A and m_B connected by a light spring on a frictionless surface. If the masses are pulled apart so that the spring is stretched after which they are released, show that the velocity of A is given by $\vec{V}_A = -\vec{V}_B \left(\frac{m_B}{m_A} \right)$ (8 marks)
- b) Given that the acceleration due to gravity on the surface of the moon is 1.7ms^{-2} and the radius of the moon is $1.70 \times 10^6 \text{m}$. Calculate the mass of the moon. (6 marks)
- c) Find the constant force needed to accelerate a mass of 10kg moving along a straight line from a speed of 54km/h to a speed of 108km/h in 5 seconds. (6 marks)

QUESTION THREE (20 MARKS)

A particle of mass m moves in the xy -plane so that its position vector is given by $\underline{r} = a \cos \omega t \underline{i} + b \sin \omega t \underline{j}$ where a, b and ω are positive constants and $a > b$

- a) Show that the:
- (i) particle moves in an ellipse (6 marks)
- (ii) force acting on the particle is always directed towards the origin (5 marks)
- b) Find the kinetic energy of the particle at points P and Q (6 marks)
- c) Find the work done by the force field in moving the particle from P to Q (3 marks)

QUESTION FOUR (20 MARKS)

- a) Find the work done in moving a particle along a straight line from $(-4, 3, 2)$ to $(2, -1, 3)$ in a force field given by $\vec{F} = 3\underline{i} - 2\underline{j} + \underline{k}$ (4 marks)

- b) An electric motor rated $3kW$ is used to lift bales of hay to a store in a dairy farm. A single bale has a mass of $5kg$. If the store is $5m$ above the ground, how many bales can the motor raise in 3 minutes? (5 marks)
- c) Given that the mass of the earth is $6 \times 10^{24} kg$ and the gravitational constant is $6.7 \times 10^{-11} m^3 kg^{-1} s^{-2}$. The radius of the earth is $6.4 \times 10^6 m$. Calculate the gravitational force on a mass of $5kg$ on the earth's surface. (5 marks)
- d) The moon revolves about the earth in $30days$. Assuming that the orbit is circular and has a radius of $4.0 \times 10^5 km$, calculate the acceleration of the moon towards the earth. (6 marks)

QUESTION FIVE (20 MARKS)

- a) A particle of mass 2 units moves in a force field \vec{F} depending on time t given by $\vec{F} = 24t^2 \underline{i} + (36t - 16)\underline{j} - 12t \underline{k}$. Assuming that at $t = 0$, the particle is located at $\vec{R}_0 = 3 \underline{i} - \underline{j} + 4 \underline{k}$ and has a velocity $\vec{V}_0 = 6 \underline{i} + 16 \underline{j} - 8 \underline{k}$, find
- (i) Velocity at any time t (5 marks)
- (ii) Position at any time t (4 marks)
- b) As an example of recoil, consider a radioactive decay in which an alpha particle, the nucleus of Helium atom of mass number 4 and atomic number 2, is emitted from a Uranium-238 nucleus originally at rest with a speed of $1.4 \times 10^7 m/s$ and kinetic energy of $4.1MeV$. Find
- (i) the velocity of the residual Thorium-234 nucleus. (6 marks)
- (ii) the kinetic energy of Thorium (5 marks)

END