(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FORTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: STA 411

COURSE TITLE: TIME SERIES ANALYSIS

DATE: 20/04/2023 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **4 Printed Pages**. *Please Turn Over.*

QUESTION 1: (30 Marks)

a) Explain the following terms as used in time series analysis:

i) Stationary process **(1mk)**
ii) Stationarity in the weak sense **(1mk)**
iii) Moving average process **(1mk)**
iv) Autoregressive process **(1mk)**
v) White noise process **(1mk)**

b) Find the autocovariance function ($\boldsymbol{\sigma(h)}$) and the autocorrelation function ($\boldsymbol{\rho(h)}$) of a moving average process of order q (MA(q)). **(8mks)**

c) Consider autoregressive process of order 1 (AR(1)) given by
$X_t = \alpha X_{t-1} + e_t$, where $\propto$ is a constant.

i) If $|\alpha| < 1$, show that $X_t$may be expressed as infinite order of a MA process. **(4mks)**

ii) Find its autocovariance function $\left(\sigma(h)\right)$ and its autocorrelation function $\left(\rho(h)\right)$. **(3mks)**

d) Transform a time series $\{X_t\}$ into another series $\{Y_t\}$ where
$Y_t = \sum_{j=-\infty}^{\infty} a_j X_{t-j}$ and $X_t = e^{i\lambda t}$
and state the changes in its amplitude, wavelength and phase angle.**(5mks)**

e) Find the spectral density function of an AR(1) process given by
$X_t = \alpha X_{t-1} + e_t$, where $|\alpha| < 1$ **(5mks)**

QUESTION 2: (20 Marks)

a) Suppose we have data up to time $\boldsymbol{n(x_1,\ x_2, \dots ,\ x_n)}$

i) Show that minimum mean squared error forecast of $\boldsymbol{x_{n+k}}$ is the conditional mean of $\boldsymbol{x_{n+k}}$ at time $\boldsymbol{n}$.
i.e. $\boldsymbol{\hat{x}(n,k) = E(x_{n+k}/x_1,\ x_2, \dots, x_n)}$ **(6mks)**

ii) Consider the AR(1) model $\boldsymbol{X_t = \alpha X_{t-1} + e_t}$, $|\propto| < \mathbf{1}$.
Forecast $\boldsymbol{x_{n+3}}$. **(2mks)**

b) Transform a moving average filter $\{\boldsymbol{X_t}\}$ into another series $\{\boldsymbol{Y_t}\}$ by the linear operator given that
$\boldsymbol{X_t = e^{i\lambda t}}$ and $\boldsymbol{Y_t = \sum_{j=-\infty}^{\infty} a_j X_{t-j}}$
Where

$$a_j = \begin{cases} \frac{1}{2m+1}, & j = 0, \mp 1, \mp 2, \dots\dots, \mp m \\ 0, & otherwise \end{cases}$$ **(12mks)**

QUESTION 3: (20 Marks)

a) Consider an AR(1) process with mean μ given by
$X_t - \mu = \alpha(X_{t-1} - \mu) + e_t, t = 1, 2, 3, \dots \dots$

Find the estimates of the parameters $\propto$ and μ using the method of least squares.
(8mks)

b) Consider a second order process AR(2) given by
$X_t = {}^1/_3 X_{t-1} + {}^2/_9 X_{t-2} + e_t$.
Show that this process is stationary and hence obtain its ACF
(12mks)

QUESTION 4: (20 Marks)

a) i) Briefly describe the main objectives in the analysis of a time series.
(3mks)

ii) State the unique feature that distinguishes time series from other branches of statistics. **(1mk)**

iii) Identify the main stages in setting up a Box-Jenkins forecasting model.
(4mks)

b) Show that the AR(2) process given by $X_t = X_{t-1} - {}^1/_2 X_{t-2} + e_t$ is stationary and hence find its ACF. **(12mks)**

QUESTION 5: (20 Marks)

a) If an observed values $(X_1, X_2, \dots, X_n)$ on a discrete time series forms $n - 1$ pairs of observation $(X_1, X_2), (X_2, X_3), \dots, (X_{n-1}, X_n)$ regarding the first observation in each pair as one variable and second observation as a second variable
Find:

i) The correlation coefficient r_1 between X_t and X_{t-1} **(5mks)**

ii) The correlation between observations at a distance k apart. **(2mks)**

b) The data below gives the average quarterly prices of a commodity for four (4) years.

Year	I	II	III	IV
1997	50.4	40.8	47.5	49.8
1998	38.3	33.6	53.2	69.5
1999	67.2	53.2	60.7	42.6
2000	55.1	56.4	61.6	65.1

Calculate the seasonal indices. **(6mks)**

c) Consider a moving average process given by $\boldsymbol{X_t = e_t + \beta e_{t-1}}$, where $(\boldsymbol{\beta_0 = 1, \beta_1 = 1})$.
Find its spectral density function. **(7mks)**