(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: MAT427/MAA415/MAA418

COURSE TITLE: NUMERICAL ANALYSIS III/NUMERICAL METHODS

DATE: 15/11/2022 **TIME:** 8 AM- 10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a. Evaluate the solution of the differential equation $\frac{dy}{dx} = y^2 + 1$ by taking four terms of the Maclaurin series for $x = 0, 0.2, 0.4$ & 0.6, given $y(0) = 0$.
(5 marks)

b. Solve $\frac{dy}{dx} = x + y$ given $y(1) = 0$ and obtain $y(1.1), y(1.2)$ by Taylor series method.
(5marks)

c. Solve numericallyy$y' = y + e^x$, y(0)=0 for $x = 0.2, 0.4$ using improved Euler method.
(5 marks)

d. Classify the following differential equations as Parabolic, Elliptic or Hyperbolic type.
   i. $u_{xx} + 2u_{xy} + u_{yy} = 0$ (2marks)
   ii. $xf_{xx} + yf_{yy} = 0, x > 0, y > 0$ (2marks)

e. Solve $u_{xx} - 2u_t = 0$ given $u(0,t) = 0, u(4,0) = 0, u(x,0) = x(4 - x)$. Assuming h=1, find the values $u$up to t=5.
(5marks)

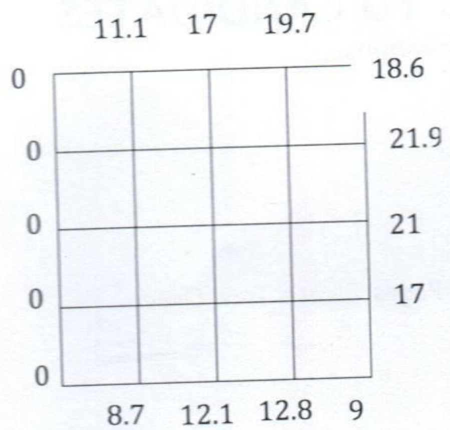
f. Using the Crank-Nich[illegible] Scheme, solve $u_{xx} = 16u_t$, $0 < x < 1, t > 1$ given

$$u(x,0) = 0, u(0,t) = 0, u(1,t) = 100t.$$

Compute u for one step in t direction taking$h = \frac{1}{4}$.
(6marks)

QUESTION TWO (20 MARKS)

Find by Liebmann's method the values at the interior lattice points of the square region of the Laplace Equation$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, whose boundary values are as given in the figure below



QUESTION THREE (20 MARKS)

Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0, y = 0, x = 3, y = 3$ $with$ $u = 0$ on the boundary and mesh length 1 unit by the following methods.

i) Direct elimination. (10marks)

ii) Gauss-Seidel method. (10marks)

QUESTION FOUR (20 MARKS)

Given the differential $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ and that $y(0) = 1$ at $x = 0.2, 0.4$ solve using:-

i) Second order Runge- Kutta method. (8marks)

ii) Fourth order Runge –Kutta method. (12mks)

QUESTION FIVE (20 MARKS)

a) Solve numerically, $4u_{xx} = u_{tt}$ with the boundary conditions $u(0, t) = 0, u(4, t) = 0$ and the initial conditions $u_t(x, 0) = 0$ and $u(x, 0) = x(4 - x),$ $taking$ $h = 1.$ (10marks)

b) Evaluate the pivotal values of the following equations taking $h = 1$ and upto one half of the period of the oscillation $16u_{xx} = u_{tt}$

given $u(0, t) = u(5, t) = 0, u(x, 0) = x^2(5 - x)$ and $u_t(x, 0) = 0$ (10marks)