(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAA 323**

COURSE TITLE: NUMERICAL ANALYSIS II

DATE: 24/11/2022 **TIME:** 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a. Derive the recurrence relation of Chebyshev polynomial of first order and find the first five polynomial equations (8 marks)

b. Evaluate $\int_0^1 \frac{1}{1+x} dx$ using Gauss Legendre (i) 2 point (ii) 3 point and (iii) 4 point formula (12 marks)

c. Find the eigenvalues and eigenvectors of the following matrix (10 marks)

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

QUESTION TWO (20 MARKS)

a. Find the first 3 non zero terms of Taylor series of the following equation $\frac{dy}{dx} = -xy$ given $y(0) = 1$ hence find $y(0.1)$ (12 marks)

b. Evaluate the integral $\int_{-1}^{1} (1 - x^2)^{\frac{3}{2}} cosx \, dx$ using Gauss Chebyshev intergral method point 1 and 2 (8 marks)

QUESTION THREE (20 MARKS)

a. Compute the eigenvalues and eigenvectors of $A = \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix}$ (8 markS)

b. Proof that $T_0(x) = 1$ and $T_1(x) = x$ in Chebyshev polynomials (5 marks)

c. Solve the following linear systems of equation using Successive over relaxation method (SOR) with $w = 1.25 \; and \; x^0 = (1,1,1)$ (3 iterations) (5 marks)

$$4x_1 + 3x_2 = 24$$

$$3x_1 + 4x_2 - x_3 = 30$$

$$-x_2 + 4x_3 = -24$$

QUESTION FOUR (20 MARKS)

a. If $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 3 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ show that $P^{-1}AP = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (10 marks)

b. Obtain the cubic spline approximation for the following set of data

X	1	2	3	4
F(x)	1	5	11	8

Hence interpolate at $Y(1.5)$ and $Y'(2)$ (15 marks)

QUESTION FIVE (20 MARKS)

a. Solve the following system of linear equations (12 marks)

$$2x_1 + x_2 + 4x_3 = 12$$
$$8x_1 - 3x_2 + 2x_3 = 20$$
$$4x_1 + 4x_2 - 3x_3 = 33$$

Using the following the decomposition methods

i. Doolittle

ii. Cholesky

b. Solve the following system of equations using Jacobi iterative method given $x_0 = y_0 = z_0 = 0$ (iterations) (8 marks)

$$4x + y + 3z = 17$$
$$x + 5y + z = 14$$
$$2x - y + 8z = 12$$