(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAP 314**

COURSE TITLE: **NUMBER THEORY**

DATE: 16/11/2022 **TIME: 8:00 AM – 10:00 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question 1: 30 marks

a) Find 17^{341}mod 5. (5 marks)

b) There are infinitely many primes. Prove (4 marks)

c) Show that an integer p is prime iff it is irreducible. (4 marks)

d) Prove that if $n \in Z$, then n^2 does not have a remainder of 2 or 3 when it is divided by 5. (8 marks)

e) Find the general solution to the following Diophantine equation $8x + 14y + 5z = 11$ (4 marks)

f) Find all the prime numbers up to 50 using sieve of Erastosthenes. (5 marks)

Question 2: 20 marks

a) Prove that $\sqrt[3]{2}$ is not a rational number. (8 marks)

b) Prove that if x is even, then $x^2 + 2x + 4$ is divisible by 4. (5 marks)

c) Prove that $2^0 + 2^1 + 2^2 + \cdots + 2^{n-1} = 2^n - 1$. (7 marks)

Question 3: 20 marks

a) The greatest common divisor of any two numbers a and b, which are not simultaneously zero, exists and is unique. It is the biggest among the common divisors of a and b. Prove. (5 marks)

b) Show that if $a \equiv b \ mod \ n$ then $b = a + nq$ for some integer q, and conversely. (4 marks)

c) Let p be a prime number and suppose that p divides ab where a and b are integers. Then either p divides a or p divides b (or both). Prove. (11 marks)

Question 4: 20 marks

a) If $n \in Z$, prove that $0.n = 0$ (5 marks)

b) Solve for x: $5x \equiv 1 mod\ 12$ (5 marks)

c) Prove that $\frac{1}{1.2} + \frac{1}{2.3} + \cdots + \frac{1}{n.(n+1)} = \frac{n}{n+1}$ for every positive integer n. (10 marks)

Question 5: 20 marks

a) Simon buys large shirts for \$ 18 each and small shirts for \$ 11 each. The shirts cost a total of \$ 1188. What is the smallest total number of shirts he could have bought? (10 marks)

b) Let p be prime, and suppose p does not divide a. then $a^{p-1} = 1(mod\ p)$. (10 marks)