(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

END OF SEMESTER EXAMINATIONS

THIRD YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND

BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MAA 312/MAT 323

COURSE TITLE: NUMERICAL ANALYSIS I

DATE: 16/11/2022 **TIME: 11:00 AM – 1:00 PM**

INSTRUCTIONS

Answer **Questions ONE** and Any other **TWO**

QUESTION ONE [30 MARKS]

a. Derive the Newton-Raphson's method and use it to find the roots of the following nonlinear polynomial equation: $f(x) = x^4 - 2x^3 + x^2 - 3x + 3 = 0$ given $x_0 = 0.5$ up to 4^{th} iteration. (10mks).

b. The following table gives the census population of a of a county for the year 1979 to 2019

Year	1979	1989	1999	2009	2019
Population (Million)	19.96	36.65	58.81	77.21	94.61

Find the rate of growth of the population in the year 2001. (5mks)

c. Using Taylor's series method, find y(0.1) correct to 4 decimal places for

$$\frac{dy}{dx} + x^2 y = 1, \quad given \ y_0 = 1, \ h = 0.1$$ (10mks)

d. From the following table find the missing value (5mks)

x	2	3	4	5	6
$f(x)$	45.0	49.2	54.1	=	67.4

QUESTION TWO [20 MARKS]

a. If $y = x^3 + x^2 - 2x + 1$, calculate the values of y for $x = 0, \ldots, 5$ and form the difference table. Find the value of y at $x = 0$ by extending the table and verify that the same value is obtained by substitution. (7mks)

b. Prove that $\Delta\nabla = \Delta - \nabla = \partial^2$ (3mks)

c. Derive the Gregory-Newton forward formula (5mks)

d. Using the Simpson's $\frac{1}{8}$ rule, compute $\int_{0.5}^{1.3} \frac{1}{1+\log x} dx$ for n=8 (5mks)

QUESTION THREE [20MARKS]

a. Convert the following to binary numbers (6mks)

$(5C2A)_{16}$

$(0.859375)_{10}$

b. Determine the root of the given equation $x^2 - 3 = 0$ for x ∈ [1, 2] (8mks)

c. *Given* $y_3 = 2,\ y_4 = -6, y_5 = 8,\ y_6 = 9$ *and* $y_7 = 17$ *calculate*

$\Delta^4 y_3$ (6mks)

QUESTION FOUR [20 MARKS]

a. Use the Lagrange interpolation to find the polynomial $p(x)$ of degree utmost two given $f(x) = 3^x$ for $x_0 = 0,\ x_1 = 1$ *and* $x_2 = 2$ (8mks)

b. Using newton divided difference formula find the values of f(2), f(8) and f(15) from the following data. (6mks)

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

c. Find $y'(1)$ for the following data points of a polynomial $y = f(x)$ (6mks)

x	0	2	4	6	8
y	4	8	15	7	6

QUESTION FIVE [20 MARKS]

a. Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ using (12mks)

i. Trapezoidal rule taking $h = \frac{1}{5}$

ii. Simpson's $\frac{1}{3}$ rule taking $h = \frac{1}{4}$

iii. Simpson's $\frac{3}{8}$ rule taking $h = \frac{1}{6}$

b. If $f(x) = x^3 - 6x^2 + 11x - 6$, find the zeros for the function by False position method taking $x_0 = 2.5,\ x_1 = 4$ (8mks)