130

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAP 424/MAA 424.**

COURSE TITLE: DIFFERENTIAL GEOMETRY

DATE: 07/09/2022 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a). Define the following terms

i). Smooth vector function (2 mks)

ii). A surface (1 mk)

iii). Cross product (1 mk)

iv). Spherical indicatrices (1 mk)

v). Bertrand Curves (1 mk)

b). Find the unit normal vector to the surface $X(u,\theta) = \langle u\cos\theta\,, u\sin\theta\,, 2\theta\rangle$ (6 mks)

c). Determine the first and the second curvature of the curve $\boldsymbol{r}(t) = \langle 2t, 4\sin t\,, 4\cos t\,\rangle$(10 mks)

d).Given the equation of the surface $X(u,v) = \langle 6u, 6u, u - v\rangle$ find its first fundamental form. (4 mks)

e). Show that the angle between the two parametric curves of a surface $X = X(u,v)$ is given by $\theta = \cos^{-1}\left(\frac{F}{\sqrt{EG}}\right)$ (4 mks)

QUESTION TWO (20 MARKS)

a). Find the area of a parallelogram whose vertices are $(-4,3), (-1,7), (3,3), (6,7)$. (3 mks)

b). Determine the unit tangent vector to $\boldsymbol{r}(t) = \langle t, e^t, -3t^2\rangle$ at $t = 0$. (4 mks)

c). Find the unit binomial vector to the curve $\boldsymbol{X}(t) = \langle 2t + 2t^3, 3t + \frac{t^2}{2}, 4t^2\rangle$ at $t = 1$. (7 mks)

d). Let $X = X(u,v)$ be surface with directions given in parametric form as $(du: dv)$ and $(\delta u, \delta v)$ whose tangential vectors are $dX = X_u du + X_v dv$ and $\delta X = X_u \delta u + X_v \delta v$ respectively. Prove that the angle between the two directions is given by

$$\theta = \cos^{-1}\left(\frac{I(d,\delta)}{\sqrt{I(d)}\sqrt{I(\delta)}}\right)$$

Where $I(d,\delta)$ is the first fundamental form of a surface. (6 mks)

QUESTION THREE (20 MARKS)

a). Determine the arc length of the curve $\boldsymbol{X}(t) = \langle 2t, 3\cos 2t\,, 3\sin 2t\,\rangle$ for $0 \le t \le \sqrt{10}$ (5 mks)

b). Find the equation of the rectifying plane of $\boldsymbol{X}(t) = \langle t^2, 1 + 4t, t^2\rangle$ at $t = 1$. (5 mks)

c). State and prove the Frenet – Serret formulas to the curve $X = X(s)$. (10 mks)

QUESTION FOUR (20 MARKS)

a). Let γ be a curve lying on the surface $\boldsymbol{X} = \boldsymbol{X}(u, v)$ where $u = u(t), v = v(t), a \leq t \leq b$. Prove that the length of the arc on the curve is given by $\int_a^b \sqrt{I}\, dt$ where I is the first fundamental form of a surface. (6 mks)

b). Find the equation of the parametric equation normal line and standard equation of the rectifying plane to the curve $\boldsymbol{X}(t) = \langle t - 1, -t^2, t^3 + 1\rangle$ at $t = 1$. (7 mks)

c). State and derive the second fundamental form of a surface $\boldsymbol{X} = \boldsymbol{X}(u, v)$ of class $C \geq 2$.

QUESTION FIVE (20marks)

a). Determine the lines of curvature to the helicoid $r(s, t) = \langle s\cos t, s\sin t, bt\rangle$. (11 mks)

b). Find the parametric equation of the tangent line to $\boldsymbol{X}(t) = \langle t, \frac{1}{2}t^2, 8t\rangle$ at $t = 1$. (3 mks)

c). Consider a parametrized surface $X(u, v) = \langle \cos u \sin v, \sin u \sin v, \cos v\rangle$ for $(u, v) \in [0,2\pi) \times [0, \pi]$. Determine the length of the curve $\left(u(t), v(t)\right) = \left(t, \frac{\pi}{2}\right)$ for $0 \leq t \leq 2\pi$ lying on the surface $X(u, v)$. (6 mks)