KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER
SPECIAL/SUPPLEMENTARY EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE (PHYSICS)

COURSE CODE: SPC 321

COURSE TITLE: QUANTUM MECHANICS I

DATE: 21/11/2022 **TIME:** 11:00AM-1:00PM

INSTRUCTIONS TO CANDIDATES

TIME: 2 HOURS

Answer question ONE and any TWO of the remaining

KIBU observes ZERO tolerance to examination cheating

QUESTION ONE [30 MARKS]

a) If $\widehat{A} = 3x^2$ and $\widehat{B} = \frac{d}{dx}$. Show that $\widehat{A}$ and $\widehat{B}$ do not commute. [4 marks]

b) Show that the function $\psi(x) = cxexp\left(-\frac{1}{2}x^2\right)$ is an eigen function of the operator $\left(x^2 - \frac{d^2}{dx^2}\right)$ and hence the corresponding eigen value. [5 marks]

c) Find the probability that a particle trapped in a box L wide can be found between o.45L and 0.55L for ground state. [4 marks]

d) A particle has a 1-dimensional wave function given by:- $\psi(x) = \sqrt{\frac{2}{L}}sin\frac{n\pi x}{L}$. Find the expectation value of x in the interval $[0, L]$. [4 marks]

e) The Hamiltonian of a simple harmonic oscillator is given by $\widehat{H} = \frac{\widehat{p}^2}{2m} + \frac{1}{2}m\omega\widehat{x}^2$ prove that: $[\widehat{p}_x, \widehat{H}] =- i\hbar m\omega^2 x$. [4 marks]

f) Write down the two Heisenberg uncertainty relations involving energy and momentum. Hence estimate the kinetic energy in MeV of a neutron confined to a nucleus of diameter $10fm$. [5 marks]

g) Show that the wave functions $\psi_n(x) = \sqrt{\frac{2}{L}}sin\frac{n\pi x}{L}$ and $\psi_m(x) = \sqrt{\frac{2}{L}}sin\frac{m\pi x}{L}$ are orthogonal. [4 marks]

QUESTION TWO [20 MARKS]

a) What boundary conditions do wave functions obey? [2 marks]

(b) A particle confined to a one dimensional potential well has a wave function given by:-

$$\psi(x) = \begin{cases} 0 & for\ x < -L/2 \\ Acos\left(\frac{3\pi x}{L}\right) & for\ -L/2 \leq x \leq L/2 \\ 0 & for\ x > -L/2 \end{cases}$$

(i) Sketch the wave function $\psi(x) \approx cos\left(\frac{3\pi x}{L}\right)$ [3 marks]

(ii) Calculate the normalization constant A [5 marks]

(iii) Calculate the probability of finding the particle in the interval $-L/4 \leq x \leq L/4$ [5 marks]

(iv) Using the Schrödinger equation $\left(-\frac{\hbar^2}{2m}\right)\left(\frac{d^2\psi}{dx^2}\right) = E\psi$ show that the energy E corresponding to this wave function is given by:- $\frac{9\pi^2\hbar^2}{2mL^2}$. [5 marks]

QUESTION THREE [20 MARKS]

a) Show that the momentum operator $-i\hbar\frac{\partial}{\partial x}$ is a Hermitian operator. Hence obtain Eigen function and Eigen values of $\hat{p}_x$. [5 marks]

b) A particle moving in one dimension is in a stationary state whose wave function is given by:- [15 marks]

$$\psi(x) = \begin{cases} 0 & for\ x > -a \\ A\left(1 + cos\frac{\pi x}{a}\right) & for -a \leq x \leq a \\ 0 & for\ x > a \end{cases}$$

Find the magnitude of A so that $\psi(x)$ is normalized, Evaluate Δx and Δp hence verify that $\Delta x \Delta p \geq \frac{\hbar}{2}$.

QUESTION FOUR [20 MARKS]

a) The one- dimensional time-independent Schrödinger equation is given by $\left(-\frac{\hbar^2}{2m}\right)\left(\frac{d^2\psi(x)}{dx^2}\right) + U(x)\psi(x) = E\psi(x)$ give the meaning of the symbols in this equation. [4 marks]

b) A particle of mass m is contained in a one-dimensional box of width a. the potential energy is infinite at the walls of the box $x = 0$ and $x = a$ and zero in between $0 < x < a$. Solve the Schrödinger equation for this particle and hence show that the normalized solutions have the forms $\psi_n(x) = \sqrt{\frac{2}{L}}sin\frac{n\pi x}{a}$ and $E_n = \frac{h^2n^2}{8ma^2}$. [12 marks]

c) For $n = 3$, find the probability that the particle will be located in the region $a/3<x<2a/3$. [4 marks]

QUESTION FIVE [20 MARKS]

a) Find the expectation values of kinetic energy, potential energy and total energy of hydrogen atom in ground state for $\psi_0 = \frac{e^{-r/a_0}}{\sqrt{\pi a_0^3}}$ where a_0 is the Bohr's radius. [8 marks]

b) Using $\frac{\partial\psi}{\partial t} = -\frac{\hbar}{2mi}\nabla^2\psi + \frac{V}{i\hbar}\psi$ and $\frac{\partial\psi^*}{\partial t} = \frac{\hbar}{2mi}\nabla^2\psi^* + \frac{V}{i\hbar}\psi^*$ show that $J_x = v_x|A|^2$. [12 marks]