(30)

(32)

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE AND

BACHELOR OF EDUCATION

COURSE CODE: MAA 314/MAA 315

COURSE TITLE: METHODS I /ANALYTICAL APPLIED MATHEMATICS

DATE: 14/11/2022 **TIME: 11:00 AM – 1:00 PM**

INSTRUCTIONS

Answer **Questions ONE** and Any other **TWO**

QUESTION ONE (30 MARKS)

a. Let f(x) be a 2π-periodic function such that $f(x) = x^2$ for $x \in [-\pi, \pi]$. Find the Fourier series for the parabolic wave. **(8mks)**

b. Prove that $J_0'(x) = -J_1(x)$ (5mks)

c. Using Rodrigues's formula derive the first four terms (5mks)

d. State whether the following functions are even or odd (3mks)

i. $f(x) = \sin x; \quad \frac{-\pi}{2} < x < \frac{\pi}{2}$

ii. $f(x) = \cos x; \quad \frac{-\pi}{2} < x < \frac{\pi}{2}$

e. Evaluate $\int_0^\infty x^3 e^{-4x} dx$ using special functions (4mks)

f. Evaluate the Bessel function $J_0(x)$ and $J_1(x)$ when x=1, correct to 3 decimal places (5mks)

QUESTION TWO (20 MARKS)

a. classify the following equation (4mks)

i. $2U_{xx} - 4U_{xy} + 2U_{yy} = 0$

ii. $9U_{xx} + 20U_{xy} + 5U_{yy} = 0$

b. Using direct integration to solve the equation $\frac{\partial^2 U}{\partial x^2} = \sin(x + y)$ given that at $y = 0$, $\frac{\partial U}{\partial x} = 1$ and at $x = 0$, $U = (y-1)^2$ (6mks)

c. Use the method of separation of variables to solve $U_x - 2U_t = U$ hence show that $U(x, 0) = 6e^{-3t}$ (10mks)

QUESTION THREE (20 MARKS)

a. Use special function to evaluate $\int_0^1 x^5 (1-x)^6 dx$ (4mks)

b. Suppose $f(t) = 0$ for $t < 0$ and that $a > 0$, show that $\mathrm{L}\{f(t-a)\} = e^{as} F(s)$ (4mks)

c. Show that $\mathrm{L}\{\cos at\} = \frac{s}{s^2 + a^2}$ (5mks)

c. Solve the initial value problem $y' - 5y = -e^{-2t}$, y(0)=3 (7mks)

QUESTION FOUR (20 MARKS)

a. Prove that $\Gamma(1) = \Gamma(2)$ (3mks)

b. Prove that $B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$ (3mks)

c. Express $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials (6mks)

d. Evaluate $\int_0^{\infty} x^6 e^{-2x} dx$ (8mks)

QUESTION FOUR (20 MARKS)

a. Give the definition of an ordinary point and state whether the following equations have an ordinary or singular point (8mks)

i. $x^2y'' + (x^2 + x)y' - y = 0$

ii. $x^2y'' + (1 + 2x)y' = 0$

b. Solve $x^2y'' + 5xy' + (3 - x)y = 0$ using the method of Frobenius. (12mks)