(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

COURSE CODE: **MAP 411**

COURSE TITLE: TOPOLOGY

DATE: 14/11/22 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a. What are the basic properties of open balls (5 marks)

b. Show that in a metric space X, a subset $Z \subset X$ is closed if and only if for every sequence p_1, p_2, ...$\in Z$ that converges to a point $p \in X$, we have $p \in Z$. (8marks)

c. Define the following
   i. Closed subset (2marks)
   ii. Cauchy sequence (2marks)
   iii. Complete metric space (1marks)

d. Show that a function $f: X \to Y$ is continuous if and only if for all open sets $U \subset X$, the preimage $f^{-1}(U) \subset X$ is open (12marks)

QUESTION TWO (20 MARKS)

a. Define the following
   i. Boundary of a subset (2marks)
   ii. Interior of a subset (2marks)

b. Show that a function $f: X \to Y$ is continuous if and only if for all basis element $B \subset Y$ for the topology on Y, $f^{-1}(B) \subset X$ is open (4 marks)

c. Show that a function $f: X \to Y$ is continuous if for every point in X , there is an open set of X on which f is a function (6marks)

d. Show that $\bar{A}$ = A∪{limit points of A} (6marks)

QUESTION THREE (20 MARKS)

a. Define the following
   i. Dense subset (1marks)
   ii. Embedding (3 marks)
   iii. Hausdorff topology (2marks)
   iv. Homeomorphism (2marks)

b. Show that if X is Hausdorff, then every sequence converges to at most one limit. (6marks)

c. Show that a map $f: Z \to \prod X_i$ is continuous if the component $f_i: Z \to X_i$ is continuous for all i. (6marks)

QUESTION FOUR (20 MARKS)

a. Define the following
   i. Topological space (1marks)
   ii. Discrete topology (1marks)
   iii. Finer topology (2marks)

iv. Basis (3marks)

b. What are the basic properties of topology (3 marks)

c. Show that the topology generated by a basis B is indeed a topology (10marks)

QUESTION FIVE (20 MARKS)

a. Define the following
   i. Order topology (4marks)
   ii. Bounded metric space (1marks)
   iii. Subspace topology (2mark)
   iv. Product topology (2marks)

b. Show that T_A is indeed a topology on A. Further more if B is a basis for T_X then {B∩A:B∈ B} is a basis for T_A (6marks)

c. Let B be a basis. Show that open sets of T are all unions of sets in B (5marks)