(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAA 414**

COURSE TITLE: **FLUID MECHANICS II**

DATE: 14/11/2022 **TIME: 11.00 AM -1.00 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) State the Buckingham's π-theorem. (2mks)

b) Differentiate between the following terms as used in dimensional analysis (6mks)

i) Geometrical symmetry

ii) Kinematic similarity

iii) Dynamic similarity

c) A source and a sink of equal strength are placed at the points $\left(\pm\frac{1}{2}c, 0\right)$ within a fixed circular boundary $y^2 + x^2 = a^2$. Show that the streamlines are given by

$(r^2 - \frac{1}{4}a^2)(r^2 - 4a^2) - 4a^2y^2 = ky(r^2 - a^2)$. (6 marks)

d) Find the equations of the streamlines due to uniform line source of strength *m* through the points A(-c,0) B(c,0) and uniform line sink of strength *2m* through the origin (6mks)

e) Show that the velocity vector **q** is everywhere tangent to the line in the xy plane along which $\psi(x, y) = constant$. (5mks)

f) Discuss the flow due to a uniform line doublet at 0 of strength μ per unit length and its axis being along OX. (5mks)

QUESTION TWO (20 MARKS)

a) State the theorem of Blasius. (2 marks)

b) Verify that $W = iK \log \{\frac{z-ia}{z+ia}\}$ is the complex potential of a steady flow of liquid about a cylinder the plane $y = 0$ being a rigid boundary. Find the forces exerted by the liquid on unit length of the cylinder. (7 marks)

c) Find the image of a line source in a circular cylinder. (5 marks)

d) Show that the velocity potential $\phi = \frac{c}{2}(x^2 + y^2 - 2z^2)$ satisfies the Laplace equation and determine its streamlines. (6 marks)

QUESTION THREE (20 MARKS)

a) The pressure difference ΔP in a pipe of diameter D and length l due to turbulent flow depends on the velocity V, viscosity μ, density ρ, roughness K. Using Buckingham's π-theorem, obtain an expression for ΔP. (7mks)

b) The resistance force R of a supersonic plane during flight can be considered as dependent upon the length of the aircraft l, velocity v, air viscosity μ, air density ρ and bulk modulus of air k. Find an expression for R. (6mks)

c) Using Rayleigh's technique, show that the resistance (R) to the motion of a sphere of diameter (D) moving with a uniform velocity (V) through a real fluid having density ρ and viscosity μ is given by $R = \rho D^2 V^2 f\left(\frac{\mu}{\rho V D}\right)$. (7mks)

QUESTION FOUR (20 MARKS)

a. Define the following terms. (3mks)
   (i) Potential flow
   (ii) Sink
   (iii) Source

b. A source of strength $10 m^2/s$ is located at (-1, 0) and a sink of strength $20 m^2/s$ is located at (1, 0). Find the velocity and stream function at P(1,1). If the dynamic pressure at infinity is zero for density of $2 Kg/m^3$, calculate the dynamic pressure at P. (5mks)

c. Show that for a two dimensionally axially symmetric boundary layer flow.

$$\int_0^{\infty}\left(1-\frac{u}{U}\right)^2\frac{r}{a}\,dn = \delta_1 - \delta_2,\quad \int_0^{\infty}\left(1-\frac{u}{U}\right)^3\frac{r}{c}\,dn = \delta_1 - 3\delta_2 + \delta_3$$

Where n the normal distance from the surface of the body is, r is the axial distance and a is the reference radius which maybe a function of the axial distance. (5mks)

d. The velocity potential function for a two dimensional flow is $\Phi = x(2y-1)$. At a point P(4, 5) determine;
   i) The velocity (3mks)
   ii) The value of the stream function (4mks)

QUESTION FIVE (20 MARKS)

a. Show that for an incompressible steady flow with constant velocity components

$$u(y) = y\frac{U}{h} + \frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\frac{y}{h}\left(1-\frac{y}{h}\right)$$

$$v = w = 0$$

Satisfy the equation of motion, when the body force is neglected $h, U, \frac{dp}{dx}$ are constants and $P = P(x)$. (7mks)

b. In the region bounded by a fixed quadrantal arc and its radii, deduce the motion due to a source and an equal sink situated at the ends of one of the bounding radii. Show that the stream line leaving either end at an angle α with the radius is

$r^2 \sin(\alpha + \theta) = x^2 \sin(\alpha - \theta)$. (7mks)

c. A vortex of circulation $2\pi k$ is at the point $z = na$ $(n > 1)$ in the presence of a plane circular boundary $Z = a$, around which there is a circulation $2\pi \lambda k$. Show that

$\lambda = \frac{1}{(n^2 - 1)}$ (6mks)