(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAP 323

COURSE TITLE: RING THEORY

DATE: 24/11/2022 TIME: 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

Question 1 (30 marks)

a) Explain the meaning of the following terms as used in ring theory.
   i) Nilpotent element in a ring
   ii) Idempotent element in a ring
   iii) Principal Ideal Domain
   iv) A commutative ring (8 marks)

b) State any three examples of noncommutative rings (3 mks)

c) Show that a Boolean ring $\mathcal{B}, x^2 = x$ for each $x \epsilon \mathcal{B}$ implies $2x = 0$ (3 mks)

d) Let x be a non zero element of a ring R with unity. Suppose there exists a unique $y \epsilon R$ such that $xyz = x$, show that $xy = 1 = yx$. (5 mks)

e) Determine the idempotents, nilpotent elements and the units of the ring of integers modulo 10 (Z_{10}) (6 mks)

f) Find all cyclic subgroups of the group of units of the ring of integers modulo 24 (Z_{24}) (5 mks)

Question two

a) Let R be the ring of real numbers with unity, and let $R[x]$ be the polynomial ring over R. Let $J = (x^2 + 1)$ be the ideal in $R[x]$ consisting of the multiples of $x^2 + 1$. Show that the quotient ${}^{R[x]}/_{J}$ is the field of complex numbers. (12 mks)

b) Let $f: R \to S$ be a homomorphism of the ring R into a ring S. Show that the set $\{f(a) | a \epsilon R\}$ is a subring of R (8 mks)

Question three

a) Let R be a commutative ring with identity.
   i) Show that if e is an idempotent element of R, then $1 - e$ is also idempotent. (6 mks)
   ii) Show that if e is an idempotent element of R then $R \cong Re \oplus R(1 - e)$ (14 mks)

Question four

a) Show that the ring of Gaussian integers $R = \{m + n\sqrt{-1} \, | m, n \, \epsilon Z\}$ is a Euclidean ring if we set $\phi(m + n\sqrt{-1}) = m^2 + n^2$ (12 mks)

b) Let A *and* B be ideals in R such that $B \subseteq A$. Show that $R / A \cong (R / B) / (A / B)$ (8 mks)

Question five

a) Find $q(r)$ *and* $r(x)$ in $Z_5[x]$ if $g(x) = 2x^3 + 3x^2 + 4x + 1$ is divided by $f(x) = 3x + 1$. (8 mks)

b) Determine whether or not the following polynomials are irreducible over Z_5
   i) $f(x) = x^3 + 2x^2 - 3x + 4$ (6 mks)
   ii) $g(x) = x^2 + 3x + 4$ (6 mks)