KIBABII UNIVERSITY

SUPPLEMENTARY/SPECIAL UNIVERSITY EXAMINATIONS

ACADEMIC YEAR 2021/2022

THIRD YEAR FIRST SEMESTER EXAMINATIONS

FOR THE DEGREE OF

BACHELOR OF SCIENCE

COURSE CODE: SPC 313

COURSE TITLE: MATHEMATICAL PHYSICS I

DATE: 18/11/2022 **TIME: 8:00AM-10:00AM**

INSTRUCTIONS TO CANDIDATES

Answer question ONE and any TWO of the remaining.

Time: 2 hours

KIBU observes ZERO tolerance to examination cheating

QUESTION ONE (30 MARKS)

a) If **A** and **B** are irrotational, prove that **A** x **B** are solenoidal (3 marks)

b) Find the angle between the surfaces $x_1 + y_2 + z_3 = 1$ and $z = x^2 + y^2 - 1$ at the point (1, +1, -1) (4 marks)

c) Show that the divergence of an inverse square force is zero (3 marks)

d) Find the volume V of the tetrahedron with vertices at the points A(1, 0, 2), B(3, -1, 4), C(1, 5, 2) and D(4, 4, 4) (4 marks)

e) Given

$$A = \begin{bmatrix} 1 & 2-i \\ 2+i & -3 \end{bmatrix},$$

Show that the eigenvalues are real and that the eigenvectors for different eigenvalues are orthogonal. (4 marks)

f) What are the eigenvalues of the matrix (3 marks)

$$M = \begin{pmatrix} 1 & 0 & -i \\ 0 & 2 & 0 \\ i & 0 & -1 \end{pmatrix} ?$$

g) Calculate the scalar product of two vectors **p** and **q** defined by

$$\mathbf{p} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}} \qquad \mathbf{q} = 5\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$

and comment on the result (3 marks)

h) Evaluate $\int_c \mathbf{A} \cdot d\mathbf{r}$ from the point P(0, 0, 0) to Q(1, 1, 1) along the curve $r = \hat{i}t + \hat{j}t^2 + \hat{k}r^3$ with $x = t, y = t^2, z = t^3$ where $A = y\hat{i} + xz\hat{j} + xyz\hat{k}$ (3 marks)

i) Show that $\int_s \mathbf{A} \cdot d\mathbf{s} = \frac{12}{5}\pi R^2$, where S is a sphere of radius R and $A = \hat{i}x^3 + \hat{j}y^3 + \hat{k}z^3$ (3 marks)

QUESTION TWO (20 MARKS)

a) Verify Stoke's theorem for the vector field $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$ over the closed contour C enclosing the plane surface S shown in Figure 1 below. AB is the arc of circle of radius 2 with its centre at the origin (10 marks)

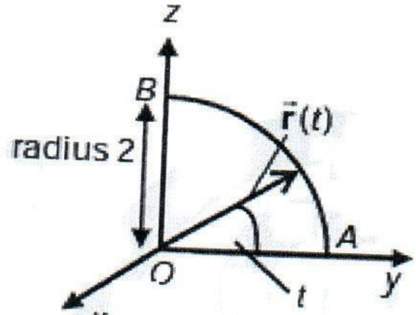

Figure 1

b) Find the eigenvalues and normalized eigenvectors of the matrix

$$\begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & 0 \\ 5 & 0 & 3 \end{pmatrix}$$

Are the eigenvectors orthogonal? Comment on this. (10 marks)

QUESTION THREE (20 MARKS)

a) Below are given sets of matrices:

$$A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, C = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, D = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

What is the effect when A, B, C and D act separately on the position vector $\begin{pmatrix} x \\ y \end{pmatrix}$?

(10 marks)

b) The figure 2 below shows a tetrahedron of vertices A(1, 0, 2), B(3, -1, 4), C(1, 5, 2) and D(4, 4, 4)

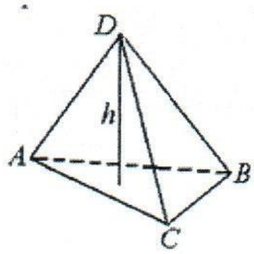

Figure 2

(i) Find the volume V of the tetrahedron (5 marks)

(ii) Find the height from the point D to the base ABC (5 marks)

QUESTION FOUR (20 MARKS)

a) Consider a cube with uniform density **ρ** and side **a**. The cube is placed such that its edges lie along x, y and z as shown in Figure 3.

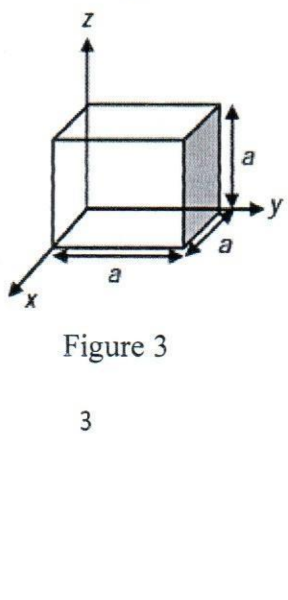

Figure 3

Determine the moment of inertia about an edge of the cube (5 marks)

b) Use divergence theorem to obtain the flux of a vector field $\vec{A} = 3x\hat{i} - y\hat{j} + 2z\hat{k}$ over a cube of side 2**a**. The vertices of the cube are at $(\pm a, \pm a, \pm a)$ as shown in Figure 4. (5 marks)

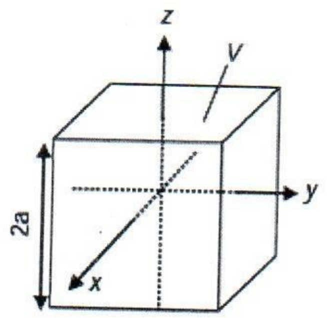

Figure 4

c) Use the divergence theorem to evaluate the flux of the vector field $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$ (5 marks)

d) Let $p, q, r, s \in \mathbf{R}$ and consider a 2 x 2 Hermitian matrix

$$A = \begin{bmatrix} p & q + ri \\ q - ri & s \end{bmatrix}$$

Compute the characteristic polynomial of A and show directly that the eigenvalues must be real numbers. (5 marks)

QUESTION FIVE (20 MARKS)

a) Figure 5 shows a parallelepiped of sides **a** and **b**

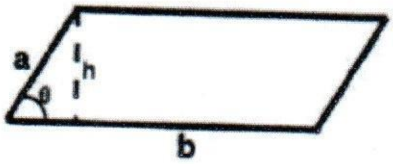

Figure 5

Given that $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{b} = 4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$, evaluate the area of the parallelepiped. (6 marks)

b) Consider the motion of a particle along a curve defined by the following parametric equations:

$$x = 2t^2 + 3 \qquad y = t^2 \qquad z = 2t$$

where t= time.

If the position vector **r** of the particle at any time **t** is expressed as $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$,

(i) Determine the speed of the particle (5 marks)

(ii) Determine the acceleration of the particle (3 marks)

c) A particle of mass **m** with position vector **r** relative to some origin O, experiences a force **F** which produces a torgue or turning effect $\mathbf{T} = \mathbf{r} \times \mathbf{F}$ about the origin O. The angular momentum of the particle about the origin O is given by $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$, where **v** is the particle's velocity. Show that the rate of change of angular momentum is equal to the applied torgue i.e.

$$\frac{d\mathbf{L}}{dt} = \mathbf{T}$$

(6 marks)