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bidual have property  $(u)$**

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## BANACH SPACES WHICH ARE $M$ -IDEALS IN THEIR BIDUAL HAVE PROPERTY (u)

by G. GODEFROY and D. LI

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### 1. Introduction.

A Banach  $X$  is an  $M$ -ideal in its bidual if the relation

$$\|y+t\| = \|y\| + \|t\|$$

holds for every  $y \in X^*$  and every  $t \in X^\perp \subseteq X^{***}$ . The spaces  $c_0(I) - I$  any set-equipped with their canonical norm belong to this class, which also contains e.g. certain spaces  $K(E,F)$  of compact operators between reflexive spaces (see [11]) and certain spaces of the form  $C(G)/C_\Lambda(G)$  where  $G$  is an abelian compact group and  $\Lambda$  is a subset of the discrete dual group (see [5]). This class has been carefully investigated, in particular by A. Lima and by the « West-Berlin school », since the notion of  $M$ -ideal was introduced by Alfsen and Effros in 1972 [1].

We will show in this paper that these spaces somehow « look like »  $c_0$ ; more precisely, that they share the property (u) with this latter space. This solves affirmatively a question that was pending for several years, and provides improvements of some results of [6] and [10].

Our proof uses non-linear arguments. The key lemma is actually a special case of a fundamental lemma ([1], lemma 1.4.) of the original article of Alfsen and Effros.

*Notation.* — The closed unit ball of a Banach space  $Z$  is denoted by  $Z_1$ , and its dual by  $Z^*$ . The topology defined on  $Z^*$  by the pointwise convergence on  $Z$  is denoted by  $\omega^*$ . The canonical injection from a Banach space  $X$  into its bidual  $X^{**}$  is denoted by  $i$ . A sequence  $(x_m)$  in  $X$  is said to be a weakly unconditionally convergent series —

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in short, w.u.c. series — if for every  $y \in X^*$ ,

$$\sum_{m=0}^{\infty} |y(x_m)| < \infty.$$

If  $(x_m)$  is a w.u.c. series, then clearly the sequence

$$s_k = \sum_{m=0}^k x_m \quad (k \geq 1)$$

is weakly Cauchy and thus it converges in  $(X^{**}, w^*)$ ; we note  $\Sigma^* x_m = \lim_{k \rightarrow \infty} (s_k)$  in  $(X^{**}, \omega^*)$ . A Banach space  $X$  has the property (u)

([14]; see [12], p. 32) if every  $z \in X^{**}$  which is in the sequential closure of  $X$  in  $(X^{**}, \omega^*)$  may be written

$$z = \Sigma^* x_m$$

for some w.u.c. series  $(x_m)$  in  $X$ .

If  $\tau: Z_1^* \rightarrow \mathbf{R}$  is a real-valued function defined on a dual unit ball  $Z_1^*$ , we denote by  $\hat{\tau}$  the smallest concave  $\omega^*$ -u.s.c. function which is greater than  $\tau$  on  $Z_1^*$ . The function  $\hat{\tau}$  is the infimum of the affine continuous functions on  $(Z_1^*, \omega^*)$  which maximize  $\tau$  on  $Z_1^*$ . The reader should consult [2] for a presentation of the basic facts about  $M$ -ideals. Similar ideas to those we use in this work are to be found e.g. in [15].

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## 2. The main result.

We will now prove:

**THEOREM 1.** — *Let  $X$  be a Banach space which is an  $M$ -ideal in its bidual. Then  $X$  has the property (u).*

*Proof.* — If  $i$  denotes the canonical injection from  $X$  into  $X^{**}$ ,  $i^{**}$  is an isometric injection from  $X^{**}$  into  $X^{****}$  which is distinct from

the canonical injection if  $X$  is not reflexive. We will use a simplified notation that we now define:  $i^*$  denotes the canonical projection from  $X^{***}$  onto  $X^*$  of kernel  $i(X)^\perp = X^\perp$ , and then of course  $i^{**}(z) = z \circ i^*$ .

From now on, we assume that  $X$  is a real Banach space. By ([2], p. 22), we can do so without loss of generality. The notation  $\mathbf{1}_{X_1^\perp}$  denotes the characteristic function of the subset  $X_1^\perp$  of the unit ball  $X_1^{***}$  of  $X^{***}$ . Thus for any  $z \in X^{**}$ ,  $(\mathbf{1}_{X_1^\perp} \cdot z \vee 0)$  denotes the supremum of 0 and of the pointwise product of  $z$  and  $\mathbf{1}_{X_1^\perp}$ .

With this notation, we have the following crucial lemma.

LEMMA 2. — *If  $X$  is an  $M$ -ideal in its bidual  $X^{**}$ , then for every  $z \in X^{**}$  and every  $t \in X_1^{***}$  one has*

$$\langle z - i^{**}(z), t \rangle = [\mathbf{1}_{X_1^\perp} \cdot z \vee 0](t) - [\mathbf{1}_{X_1^\perp} \cdot z \vee 0](-t).$$

This lemma is actually a special case of ([1], lemma I.4). For sake of completeness, we give a simplified proof of this special case.

*Proof.* — If  $\Psi$  is a function from  $X_1^{***}$  to  $\mathbf{R}^+$ , we define

$$\mathfrak{G}^-(\Psi) = \{(t, \lambda) \in X_1^{***} \times \mathbf{R}^+ \mid 0 \leq \lambda \leq \Psi(t)\}$$

we let  $\tau = (\mathbf{1}_{X_1^\perp} \cdot z \vee 0)$ , and

$$B = \{(t, z(t)) \in X_1^\perp \times \mathbf{R}^+ \mid z(t) \geq 0\}.$$

We clearly have

$$(1) \quad \mathfrak{G}^-(\hat{\tau}) = \overline{\text{conv}}^* (\mathfrak{G}^-(\tau)).$$

On the other hand,

$$(2) \quad X_1^{***} = \text{conv} (X_1^* \cup X_1^\perp) \text{ since } X \text{ is an } M\text{-ideal in } X^{**}$$

$$(3) \quad \text{if } 0 \leq \lambda \leq z(t), (t, \lambda) \in \text{conv} \{(t, 0); (t, z(t))\}.$$

From (1), (2) and (3) follows

$$\begin{aligned} \mathfrak{G}^-(\hat{\tau}) &= \overline{\text{conv}}^* (\mathfrak{G}^-(\tau)) \\ &= \text{conv} \{(X_1^{***} \times \{0\}) \cup B\} \\ &= \text{conv} \{(X_1^* \times \{0\}) \cup (X_1^\perp \times \{0\}) \cup B\} \end{aligned}$$

since by  $w^*$ -compactness we don't need to take  $w^*$ -closures.

For every  $t \in X_1^{***}$ ,  $(t, \hat{\tau}(t)) \in \mathfrak{G}^-(\hat{\tau})$ , hence we may write  $(t, \hat{\tau}(t)) = \alpha_1(t_1, 0) + \alpha_2(t_2, 0) + \alpha_3(t_3, z(t_3))$  with :

$$\begin{cases} t_1 \in X^* \\ t_2 \in X_1^\perp \\ t_3 \in X_1^\perp; & z(t_3) \geq 0 \\ \alpha_1, \alpha_2, \alpha_3 \geq 0, & \alpha_1 + \alpha_2 + \alpha_3 = 1. \end{cases}$$

Since  $t = \alpha_1 t_1 + (\alpha_2 t_2 + \alpha_3 t_3)$  is the unique decomposition of  $t$  on the direct sum  $X^* \oplus X^\perp$  one has

$$t - i^*(t) = \alpha_2 t_2 + \alpha_3 t_3.$$

Since  $z(t_3) \geq 0$ , one has  $\tau(t_3) = z(t_3)$  and  $\tau(-t_3) = 0$ . Since  $\hat{\tau}$  is concave, one has

$$\begin{aligned} \hat{\tau}(t) &= \alpha_3 z(t_3) \\ &\geq \sum_{i=1}^3 \alpha_i \hat{\tau}(t_i) \\ &\geq \sum_{i=1}^3 \alpha_i \tau(t_i) \\ &= \alpha_2 \tau(t_2) + \alpha_3 \tau(t_3) \\ &= \alpha_2 \tau(t_2) + \alpha_3 z(t_3) \end{aligned}$$

hence  $\alpha_2 \tau(t_2) \leq 0$ ; if  $\alpha_2 = 0$  we may take  $t_2 = 0$  as well; if  $\alpha_2 > 0$  this implies  $\tau(t_2) \leq 0$ , hence  $z(t_2) \leq 0$ . In both cases, we have  $z(-t_2) \geq 0$  and thus  $z(-t_2) = \tau(-t_2)$ .

Again by concavity of  $\hat{\tau}$ , one has

$$\begin{aligned} \hat{\tau}(-t) &\geq \sum_{i=1}^3 \alpha_i \hat{\tau}(-t_i) \\ &\geq \sum_{i=1}^3 \alpha_i \tau(-t_i) \\ &= \alpha_2 \tau(-t_2) + \alpha_3 \tau(-t_3) \\ &= \alpha_2 z(-t_2) \end{aligned}$$

hence

$$-\hat{\tau}(-t) \leq \alpha_2 z(t_2)$$

and therefore

$$\hat{\tau}(t) - \hat{\tau}(-t) \leq \alpha_3 z(t_3) + \alpha_2 z(t_2)$$

and

$$\begin{aligned} \alpha_3 z(t_3) + \alpha_2 z(t_2) &= \langle z, \alpha_2 t_2 + \alpha_3 t_3 \rangle \\ &= \langle z, t - i^*(t) \rangle \\ &= \langle z - i^{**}(z), t \rangle. \end{aligned}$$

Now the functions  $\Phi(t) = \hat{\tau}(t) - \hat{\tau}(-t)$  and  $(z - i^{**}(z))$  are both odd functions on  $X_1^{***}$  and they satisfy  $\Phi \leq z - i^{**}(z)$ ; hence necessarily  $\Phi = z - i^{**}(z)$  on  $X_1^{***}$ . □

We now come back to the proof of theorem 1. By lemma 2, for every  $z \in X^{**}$ , we can write

$$\forall t \in X_1^{***}, \quad \langle i^{**}(z), t \rangle = (z(t) - \hat{\tau}(t)) + \hat{\tau}(-t)$$

hence if we let

$$\begin{aligned} h_1(t) &= z(t) - \hat{\tau}(t) \\ h_2(t) &= -\hat{\tau}(-t) \end{aligned}$$

we have  $i^{**}(z) = h_1 - h_2$  and  $h_1, h_2$  are both l.s.c. on  $(X_1^{***}, w^*)$ .

We need now a topological argument for going down to  $(X_1^*, w^*)$ .

LEMMA 3 (Saint-Raymond). — *Let  $K$  be a compact topological space and  $S : K \rightarrow K'$  be a continuous surjection. Let  $f$  be a function from  $K'$  to  $\mathbb{R}$  which is such that  $(f \circ S)$  is the difference of two l.s.c. functions on  $K$ . Then  $f$  is the difference of two l.s.c. functions on  $K'$ .*

*Proof.* — Write  $f \circ S = g_1 - g_2$  where  $g_1, g_2$  are l.s.c. on  $K$ ; we define for  $y \in K'$

$$\begin{aligned} \tilde{g}_1(y) &= \inf \{g_1(t) \mid S(t) = y\} \\ \tilde{g}_2(y) &= \inf \{g_2(t) \mid S(t) = y\} \end{aligned}$$

the functions  $\tilde{g}_i (i=1,2)$  are l.s.c. on  $K'$ . Indeed, pick  $\alpha < \tilde{g}_i(y)$ ; this means

$$(1) \quad \forall t \in S^{-1}(y), \quad g_i(t) > \alpha.$$

Since  $g_i$  is l.s.c. and  $S^{-1}(y)$  is compact, (1) implies that there exists  $\varepsilon > 0$  and an open neighbourhood  $V$  of  $S^{-1}(y)$  such that

$$(2) \quad \forall t \in V, \quad g_i(t) > \alpha + \varepsilon.$$

Again by compactness, there exists a neighbourhood  $W$  of  $y$  such

that  $S^{-1}(W) \subseteq V$ ; by (2) and the definition of  $\tilde{g}_i$ , this implies

$$\forall y' \in W, \quad \tilde{g}_i(y') \geq \alpha + \varepsilon > \alpha$$

and thus  $\tilde{g}_i$  is l.s.c.

We show now that  $f = \tilde{g}_1 - \tilde{g}_2$ ; for every  $y \in K'$  and  $t \in S^{-1}(y)$ , one has

$$\begin{aligned} \tilde{g}_1(y) &\leq g_1(t) = f \circ S(t) + g_2(t) \\ &= f(y) + g_2(t) \end{aligned}$$

hence by definition of  $\tilde{g}_2$

$$\tilde{g}_1(y) \leq f(y) + \tilde{g}_2(y).$$

On the other hand,

$$\begin{aligned} f(y) + \tilde{g}_2(y) &\leq f(y) + g_2(t) \\ &= f \circ S(t) + g_2(t) \\ &= g_1(t) \end{aligned}$$

and thus by definition of  $\tilde{g}_1$ ,

$$f(y) + \tilde{g}_2(y) \leq \tilde{g}_1(y)$$

and this concludes the proof of lemma 3. □

Let us now conclude the proof of the theorem. Since

$$i^{**}(z) = z \circ i^* = h_1 - h_2$$

with  $h_1$  and  $h_2$  l.s.c. on  $(X_1^{***}, w^*)$ , we may apply lemma 3 with  $f = z$ ,  $S = i^*$  and  $K' = (X_1^*, w^*)$ ; this lemma provides us with the l.s.c. functions  $\tilde{h}_1$  and  $\tilde{h}_2$  on  $(X_1^*, w^*)$  such that  $z = \tilde{h}_1 - \tilde{h}_2$ .

If now  $z = \lim_{n \rightarrow \infty} x_n$  in  $(X^{**}, w^*)$ , where  $(x_n)$  is a sequence in  $X$ , we let

$$Y = \overline{\text{span} \{x_n | n \geq 1\}}$$

and we call  $Q$  the canonical quotient map from  $X^*$  onto  $Y^*$ ; since  $z \in Y^{\perp\perp} = Q^*(Y^{**})$ , there is  $z' \in Y^{**}$  such that  $z = z' \circ Q$ ; again by lemma 3, there exist two l.s.c. functions  $\tilde{h}_1$  and  $\tilde{h}_2$  on  $(Y_1^*, w^*)$  such that

$$z' = \tilde{h}_1 - \tilde{h}_2.$$

But since  $Y$  is separable, the  $w^*$ -topology on  $Y_1^*$  is defined by a metric  $d$ , and then classically the sequences  $f_n^i (i=1,2)$  defined for  $y \in Y_1^*$  and  $n \geq 1$  by

$$f_n^i(y) = \inf \{ \tilde{h}_i(y') + nd(y, y') \mid y' \in Y_1^* \}$$

are increasing sequences of continuous functions on  $(Y_1^*, w^*)$  which converge pointwise to  $\tilde{h}_i$ . Now the sequence  $u_n (n \geq 0)$  of continuous functions on  $(Y_1^*, w^*)$  defined by

$$\begin{aligned} u_0 &= f_1^1 - f_1^2 \\ u_n &= f_{n+1}^1 + f_n^2 - f_n^1 - f_{n+1}^2 \quad (n \geq 1) \end{aligned}$$

satisfies

$$\sum_{n=0}^{\infty} |u_n(y)| < \infty, \quad \forall y \in Y_1^*$$

and

$$\sum_{n=0}^{\infty} u_n(y) = z'(y), \quad \forall y \in Y_1^*.$$

But we still have

$$z'(y) = \lim_{n \rightarrow \infty} x_n(y), \quad \forall y \in Y_1^*$$

in this situation, a classical lemma of Pelczynski [14] (see [12], p. 32), which relies on a convex combination argument, shows that there is a sequence  $(c_n)_{n \geq 0}$  in  $Y$  with

$$\sum_{n=0}^{\infty} |c_n(y)| < \infty, \quad \forall y \in Y_1^*$$

and

$$\sum_{n=0}^{\infty} c_n(y) = z'(y), \quad \forall y \in Y_1^*$$

and since  $z = Q^*(z')$  and  $c_n = Q^*(c_n)$ , this shows that

$$z = \Sigma^* c_n$$

and  $(c_n)$  is a w.u.c. series in  $X$ . □

Before mentioning a few applications of our result, we would like to mention that the proof provides an explicit expression of  $z \in X^{**}$



as a difference of two l.s.c. functions on  $(X_1^*, w^*)$ ; indeed, if we define for  $y \in X_1^*$

$$v(y) = \inf \{z(t) - [\mathbf{1}_{X_1^+} \widehat{z} \vee 0](t) \mid t \in X_1^{***}, i^*(t) = y\}$$

then the functions  $v$  and  $(v-z)$  are both l.s.c. on  $(X_1^*, w^*)$ .

### 3. Applications.

We gather in this section a few consequences of our result.

**3.1.** P. Saab and the first-named author showed in ([6], Theorem 1) that if  $X$  is an  $M$ -ideal in its bidual then  $X$  has the property (V) of Pelczynski; the proof uses «pseudo-balls» ([3]) and the local reflexivity principle. Since such an  $X$  does not contain  $\ell^1(N)$ , our result is an improvement of ([6], Theorem 1), and of course also of the fact ([10]) that non-reflexive  $M$ -ideals in their bidual contain  $c_0(N)$ .

Another result of [6] is a structural result (Corollary 6) for certain spaces  $E$  such that  $K(E)$  is an  $M$ -ideal in  $L(E)$ . The proof uses Banach algebras techniques that require to work with complex Banach spaces. This is not needed any more, and our result together with the proofs of ([6], Theorem 4 and Corollary 6) implies for instance the

**PROPOSITION 4.** — *Let  $E$  be a separable reflexive space with A.P. such that  $K(E)$  is an  $M$ -ideal in  $L(E)$ . Then  $E$  is complemented in a reflexive space with an unconditional finite dimensional decomposition.*

There are some similarities between the techniques of [6] and of the present work; the main difference is that instead of using l.s.c. affine functions on a non-symmetric convex set — namely, the state space of a Banach algebra — we employ l.s.c. convex functions on a symmetric convex set — namely, a dual unit ball.

**3.2.** A Banach space  $Y$  is said to have to property (X) [7] if the following holds:  $z \in Y^{**}$  belongs to  $Y$  if and only if for every w.u.c. series  $(y_n)$  in  $Y^*$ ,

$$z(\Sigma_* y_n) = \Sigma z(y_n)$$

where  $(\Sigma_* y_n)$  denotes the limit of the sequence  $\left\{s_k = \sum_{n=1}^k y_n \mid k \geq 1\right\}$  in  $(Y^*, w^*)$ . This condition roughly means that an abstract Radon-

Nikodym theorem is available for deciding which elements of  $Y^{**}$  belong to  $Y$ . Property (X) is equivalent to saying that  $Y < \ell^1(N)$  for Edgar's ordering of Banach spaces [4]. For more details about this property, the reader may consult the recent survey [8].

Let us recall now the following easy

*Claim.* — If  $X$  is separable, does not contain  $\ell^1(N)$  and has the property (u), then  $X^*$  has the property (X).

*Proof of the claim.* — We must show that every  $t \in X^{***}$  such that  $t(\Sigma_* z_n) = \Sigma t(z_n)$  for every w.u.c. series in  $X^{**}$  belongs to  $X^*$ . We can write  $t = y + t'$  with  $y \in X^*$  and  $t' \in X^\perp$ ; since  $y(\Sigma_* z_n) = \Sigma y(z_n)$  by  $w^*$ -continuity of  $y$ , we also have  $t'(\Sigma_* z_n) = \Sigma t'(z_n)$ .

Since  $X$  is separable, does not contain  $\ell^1(N)$  and has (u), every  $z \in X^{**}$  can be written  $z = \Sigma_* x_n = \Sigma_* i(x_n)$  for some w.u.c. series in  $X$ ; but since  $t'(x_n) = 0$  for every  $n$ , this implies  $t'(z) = 0$ , hence  $t' = 0$  and  $t = y \in X^*$ .

Now this claim, together with theorem 1, shows :

**PROPOSITION 5.** — *If a separable Banach space  $X$  is an  $M$ -ideal in its bidual, then  $X^*$  has the property (X).*

By ([8], Theorem VII.8) this implies the following

**COROLLARY 6.** — *Let  $X$  be a separable Banach space  $X$  which is an  $M$ -ideal in its bidual, and let  $Y$  be an arbitrary Banach space. Let  $T : X^{**} \rightarrow Y^*$  be a bounded linear operator. The following are equivalent :*

- (1) *there is an operator  $T_0 = Y \rightarrow X^*$  such that  $T_0^* = T$ ,*
- (2)  *$\ker(T)$  and  $T(X_1^{**})$  are  $w^*$ -closed,*
- (3)  *$T$  is  $(w^* - w^*)$ -Borel,*
- (4)  *$T$  is  $(w^* - w^*)$ -strongly Baire measurable.*

Let us conclude this work with a few natural questions.

**Question 3.4.** — *Does there exist a separable Asplund space with property (u) which is not isomorphic to an  $M$ -ideal in its bidual? It looks reasonable to believe that this question has a positive answer; a candidate example is the space  $K(L^p)$  ( $1 < p < \infty, p \neq 2$ ) which has (u) ([12], Th. 3) but is not  $M$ -ideal of its bidual for its canonical norm [11].*

Let us also mention that a separable  $\mathcal{L}^\infty$ -space which is isomorphic to an  $M$ -ideal in its bidual is in fact isomorphic to  $c_0(N)$  [9]. We do not know whether any isomorphic predual of  $\ell^1(N)$  which has property (u) is isomorphic to  $c_0(N)$ .

*Question 3.5.* — A reformulation of Proposition 5 is that if  $Y$  is a separable space such that there exists a projection  $\pi: Y^{**} \rightarrow Y$  with :

(a)  $\|z\| = \|\pi(z)\| + \|z - \pi(z)\|, \forall z \in Y^{**}$

(b)  $(\ker \pi)$   $w^*$ -closed,

then  $Y$  has the property (X). It is not known whether the assumption (b) can be removed, or whether (a) alone implies the weaker property (V\*) (see [14]), or at least that  $Y$  contains a complemented copy of  $\ell^1(N)$  if it is not reflexive.

#### BIBLIOGRAPHY

- [1] E. M. ALFSEN, E. G. EFFROS, Structure in real Banach spaces I, *Ann. of Math.*, 96 (1972), 98-128.
- [2] E. BEHREND,  $M$ -structure and the Banach-Stone theorem, *Lecture Notes in Mathematics* 736, Springer-Verlag (1977).
- [3] E. BEHREND, P. HARMAND, Banach spaces which are proper  $M$ -ideals, *Studia Mathematica*, 81 (1985), 159-169.
- [4] G. A. EDGAR, An ordering of Banach spaces, *Pacific J. of Maths*, 108, 1 (1983), 83-98.
- [5] G. GODEFROY, On Riesz subsets of abelian discrete groups, *Israel J. of Maths*, 61, 3 (1988), 301-331.
- [6] G. GODEFROY, P. SAAB, Weakly unconditionally convergent series in  $M$ -ideals, *Math. Scand.*, to appear.
- [7] G. GODEFROY, M. TALAGRAND, Nouvelles classes d'espaces de Banach à predual unique, *Séminaire d'Ana. Fonct. de l'École Polytechnique*, Exposé n° 6 (1980/1981).
- [8] G. GODEFROY, Existence and uniqueness of isometric preduals: a survey, in *Banach space Theory*, Proceedings of a Research workshop held July 5-25, 1987, *Contemporary Mathematics* vol. 85 (1989), 131-194.
- [9] G. GODEFROY, D. LI, Some natural families of  $M$ -ideals, to appear.
- [10] P. HARMAND, A. LIMA, On spaces which are  $M$ -ideals in their biduals, *Trans. Amer. Math. Soc.*, 283-1 (1984), 253-264.
- [11] A. LIMA,  $M$ -ideals of compact operators in classical Banach spaces, *Math. Scand.*, 44 (1979), 207-217.
- [12] J. LINDENSTRAUSS, L. TZAFRIRI, *Classical Banach spaces*, Vol. II, Springer-Verlag (1979).

- [13] F. LUST, Produits tensoriels projectifs d'espaces de Banach faiblement séquentiellement complets, Coll. Math., 36-2 (1976), 255-267.
- [14] A. PELCZYNSKI, Banach spaces on which every unconditionally convergent operator is weakly compact, Bull. Acad. Pol. Sciences, 10 (1962), 641-648.
- [15] R. R. SMITH, J. D. WARD, Applications of convexity and  $M$ -ideal theory to quotient Banach algebras, Quart. J. of Maths. Oxford, 2-30 (1978), 365-384.

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