(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAA 412/MAT 421**

COURSE TITLE: PDE I

DATE: 17/11/2022 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) By eliminating the arbitrary functions, obtain the PDE from $z = f(x + ct) + g(x - ct)$(8mks)

b) Find the complete and general solution of $p + 3q = 5z + \tan(y - 3x)$ (7mks)

c) Prove the Lagrange's linear equation of the type
$Pp + Qq = R$

d) Where P, Q, R are functions of x, y, z and $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. Use the arbitrary function $f(u, v) = 0$. Where u and v are functions of x, y, z (8mks)

e) Solve the following PDE $\frac{\partial^3 z}{\partial x^2 \partial y} + 24xy^2 + \sin(3x - 2y) + e^x = 0$ (7mks)

QUESTION TWO (20 MARKS)

a) Find the solution of the equation $(x^2 - 1)p + xyq + y^2 z = x^2 - 1$ which is zero on the positive y –axis. In what region of the xy plane is the solution unique? (7mks)

b) Find the complete and general solution of $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ (7mks)

c) Solve the following equations
   i. $p - x^2 = q + y^2$ (3mks)
   ii. $p^2 + q^3 = 5$ (3mks)

QUESTION THREE (20 MARKS)

a) Find the integral surface of the linear PDE $x(y^2 + z^2)p - y(x^2 + 1)q = z(x^2 + y^2)$ which contains the straight line $x + y = 0,\ z = 1$. (6 Marks)

b) Solve $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$ where $D = \frac{\partial}{\partial x}$ $D' = \frac{\partial}{\partial y}$ (6 Marks)

c) Solve

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{-y(z^2 + x^2)} = \frac{dz}{z(x^2 + y^2)}$$

Using the multipliers $l_1 = x, m_1 = y, l_2 = \frac{1}{x}$ & $m_2 = -\frac{1}{y}$. (8 marks)

QUESTION FOUR (20 MARKS)

a) Find the integral surface of the PDE given by $z^2(p^2 + q^2) = x^2 + y^2$ (8mks)

b) Show that the equations $xp = yq, z(xp + yq) = 2xy$ are compatible and hence solve them. (12mks)

QUESTION FIVE (20 MARKS)

a) Consider an equation of the form $F(x, y, z, a, b) = 0$ where a and b denote arbitrary constants and z is a function of x and y, explain how one can a PDE from equation (4 Marks)

b) Find the complete and general solution of Lagrange equation $y^2 \frac{\partial z}{\partial x} - xy\frac{\partial z}{\partial x} = x(z - 2y)$ (6 Marks)

c) Find the Partial Differential Equation by eliminating the arbitrary constants a and b from $z = (x^2 + a)(y^2 + b)$ (5 Marks)

d) Find the complete integral of the equation $p(q^2 + 1) + (b - z)q = 0$ (5 Marks)