(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND

BACHELOR OF SCIENCE

COURSE CODE: MAA 423/MAT426

COURSE TITLE: FOURIER SERIES

DATE: 23/11/2022   **TIME:** 11:00 AM – 1:00 PM

TIME: 2 Hours

QUESTION 1 (30 MARKS)

a) Find the limit $\lim\limits_{x \to 0} \left\{ \dfrac{x^{14} - 11 + \dfrac{1}{2} x^8}{x^6} \right\}$

(5 marks)

b) Determine whether the given functions are even, odd or neither

    i) $f(x) = \cot\left(\dfrac{n\pi x}{L} \right)$ on $-L \le x \le L$

    ii) $f(x) = x^2 - 1 + e^{2x}$ on $-L \le x \le L$ (9 marks)

c) Compute the Maclaurin series as far as x^6 term for the following functions

    i) $\dfrac{\sin(x)}{x}$

    ii) $\dfrac{1 - \cos(x)}{x^2}$ (8 marks)

d) The Fourier series of the function f defined by $f(x) = x^2$ on the interval $[-\pi, \pi]$ is known to be convergent.

What do you understand by

    i)   period of f
    ii)   Fourier coefficients of expansion

Give a sketch graph of two periodic extensions of f (8 marks)

QUESTION 2 (20 MARKS)

a) Show that the equation $u_t = 8u_{xx} + u_x$ is parabolic   (7 marks)

b) Determine a transformation which makes the equation; $u_t = \alpha^2 u_{xx}$ variable separable.

(6marks)

c) Solve the heat equation $u_t = \alpha^2 u_{xx}$, $0 < x < 1$, $t > 0$ with the Dirichlet boundary conditions $u(t,0) = u(t,1) = 10$, $t > 0$ and initial conditions $u(0,x) = g(x) = x$, $0 \le x \le 1$ applying variable separation

(7 marks)

QUESTION 3 (20 MARKS)

Find the Fourier series of the function defined in pieces (piecewise constant function) by

$$f(x) = \begin{cases} 8 & 0 < x < 2 \\ -8 & 0 < x < 4 \end{cases}$$

where f is periodic with period 4. What does the series converge to at

i) $x = 0.42$

ii) $x = 3$ (20 marks)

QUESTION.4 [20 marks]

(a) Given the voltage $v = f(t)$ $volts$, and $i = F(t)$ $amperes$, such that

$v = 12.0 + 5.2\cos wt + 2.4\cos 2wt + 0.9\cos 3wt + ... + 2.7\sin wt + 1.8\sin 2wt + 0.2\sin 3wt + ...$

$i = 8.50 + 4.1\cos wt + 2.0\cos 2wt + 0.6\cos 3wt + ... + 3.6\sin wt + 1.2\sin 2wt + 0.3\sin 3wt + ...$

find the average value of power vi in $watts$, over one cycle.   [9 marks]

(b) For the function $f(x) = \begin{cases} 20x & -5 < x < 5 \\ f(x+10) & otherwise \end{cases}$

(i) sketch graph of $f(x)$ over the interval $-20 < x < 20$

(ii) state period of $f(x)$

(iii) obtain Fourier series for $f(x)$ [11 marks]

QUESTION 5 [20 marks]

One cycle of a periodic waveform $y = f(x)$ of period 2π is defined by the below data.

x^0	0	30	60	90	120	150	180	210	240	270	300	330
$y(x)$	150	200	230	240	200	80	30	40	90	120	100	110

Determine the approximate Fourier series for $y = f(x)$ up to and including the third harmonic.

[20 marks]