(Knowledge for Development)

# KIBABII UNIVERSITY

## UNIVERSITY EXAMINATIONS

## 2021/2022 ACADEMIC YEAR

## THIRD YEAR SECOND SEMESTER

## SPECIAL/SUPPLEMENTARY EXAMINATION

## FOR THE DEGREE OF BACHELOR OF EDUCATION

**COURSE CODE:** **STA 325**

**COURSE TITLE:** **MULTIVARIATE PROBABILITY DISTRIBUTION**

**DATE:** 23/11/2022 **TIME:** 8:00 AM – 10:00 AM

---

## INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

This Paper Consists of 3 Printed Pages. Please Turn Over.

## QUESTION ONE (30 MARKS)

a) Define a random vector (2mks)

b) Let three random variables have the joint probability density function (pdf) as follows:

$$f(x_1, x_2, x_3) = \begin{cases} 8x_1x_2x_3; & 0 < x_1 < 1, 0 < x_2 < 1, 0 < x_3 < 1 \\ 0, & otherwise \end{cases}$$

Compute the expected value $5X_1X_2^3 + 3x_2X_3^4$ of (6mks)

c) Let $\underline{x}$ be a trivariate random vector such that

$E(\underline{x}) = 0$ and $var(\underline{x}) = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \end{bmatrix}$.Find the expected value of the quadratic form

$$P = (x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_1)^2$$ (8mks)

(a) Let $\underline{x}$ be a random vector having the covariance matrix

$$\Sigma = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{bmatrix}$$

(i) Obtain the population correlation matrix $(\rho)$ and $V^{\frac{1}{2}}$ (8mks)

(ii) Multiply your matrices to check the relation $V^{\frac{1}{2}}\rho V^{\frac{1}{2}}$. (6mks)

## QUESTION TWO [20 MARKS]

a) Let random variables X, Y and Z have the joint pdf given by

$$f(x, y, z) = \begin{cases} \frac{12x^2 + 12yz}{7}; & 0 < x < 1, 0 < y < 1, 0 < z < 1 \\ 0, & otherwise \end{cases}$$

i) Use the joint pdf to find $f(z \backslash x, y)$. (3mks)

ii) What is the $E(Z \backslash x = \frac{1}{2}, y = \frac{1}{2})$? (5mks)

iii) Find $Var(Z \backslash x = \frac{1}{2}, y = \frac{1}{2})$. (5mks)

b) Show that the sample mean is an unbiased estimator of $\underline{\mu}$ and that the sample variance is biased estimator of matrix $\Sigma$ (7mks)

## QUESTION THREE [20 MARKS]

(a) In an experiment involving two correlated variables, the following sample statistics were obtained: $\underline{\overline{X}} = \begin{bmatrix} 10.00 \\ 10.00 \end{bmatrix}$ $S = \begin{bmatrix} 0.7986 & 0.6793 \\ 0.6793 & 0.7343 \end{bmatrix}$. Determine

(i) Principal components (8mks)

(ii) Variance of each principal component (3mks)

(iii) Percentage of variance explained by each principal component (3mks)

(b) Let $\underline{x}$ be a $p$-variate random vector, $A$ be a non-zero matrix constants and $\underline{b}$ a $p \times 1$ vector of constants, show that

$$var(A\underline{x} + \underline{b}) = A\Sigma A'$$ (6mks)

## QUESTION FOUR [20 MARKS]

a) Let $\underline{x} = [5,1,3]$ and $\underline{y} = [-1,3,1]$. Find

(i) The length of $\underline{x}$ (2mk)

(ii) The angle between $\underline{x}$ and $\underline{y}$ (3mks)

(iii) The length of the projection of $\underline{x}$ on $\underline{y}$ (2mk)

b) Consider the following $n = 3$ observations on $p = 2$ variables

Variable 1: $x_{11} = 2, x_{21} = 3, x_{31} = 4$

Variable 1: $x_{12} = 1, x_{22} = 2, x_{32} = 4$

(i) Compute the sample means $\bar{x}_1$ and $\bar{x}_2$ and the sample variances $S_{11}$ and $S_{22}$ (6mks)

(ii) Compute the sample covariance $S_{12}$ and the sample correlation coefficient$r_{12}$ and interpret these quantities (4mks)

(iii) Display the sample mean array $\bar{x}$, the sample correlation array $R$ and the sample variance-covariance $S_n$ (3mks)

## QUESTION FIVE [20 MARKS]

d) Define the following terms

(i) Random vector (2mks)

(ii) Positive definite matrix (2mks)

e) Assume $\underline{x}' = (x_1, x_2, x_3)$ is normally distributed with mean vector $\underline{\mu} = (1, -1, 2)$ and variance matrix $\Sigma = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{bmatrix}$. Find the distribution of $3x_1 - 2x_2 + x_3$ (8mks)

f) Find the maximum likelihood estimators of the mean vector $\underline{\mu}$ and covariance matrix $\Sigma$ based on the data matrix

$$x = \begin{bmatrix} 5 & 1 \\ -2 & 3 \\ 3 & 4 \end{bmatrix}$$

(8mks)