(Knowledge for Development)

**KIBABII UNIVERSITY**

**UNIVERSITY EXAMINATIONS**

**2021/2022 ACADEMIC YEAR**

**THIRD YEAR SPECIAL/SUPPLEMENTARY EXAMINATION**

**FOR THE DEGREE OF BACHELOR OF SCIENCE**

**(RENEWABLE ENERGY)**

**COURSE CODE:** **MAT 351**

**COURSE TITLE:** **COURSE TITLE: ENGINEERING MATHEMATICS III**

**DATE:** 18/11/2022 **TIME: 2 PM -4 PM**

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## INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

**QUESTION ONE (30marks)**

(a). Define the following terms

(i). Laplace transform of a function $f(t)$ **(2 Mark)**

(ii). Step function a function **(1 Mark)**

(iii). Laplace operator **(2 Mark)**

(iv). Periodic function **(1 Mark)**

b). Find the $\boldsymbol{div}\ (\boldsymbol{curl}\ \boldsymbol{v})$ given that $\boldsymbol{v} = x^2\boldsymbol{i} - 2xy^2\boldsymbol{j} + 2z\boldsymbol{k}$. **(5 Marks)**

c). Find the inverse Laplace transform of $\frac{4s^2+12}{s(s^2+4)}$ **(5 Marks)**

d). Find the directional derivative of $\phi = x^2y + zx$ at (1,1,1) in the direction of $u = -\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$. **(5 Marks)**

e). (i). Show that the following limit does not exists

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{x^2+y^2}$$ **(4 Marks)**

f). Find the Fourier sine series for $f(x) = x^2$ on $0 \le x \le L$ **(5 Marks)**

**QUESTION TWO (20 MKS)**

a). When are two functions said to be orthogonal? **(1 Mark)**

b). Find the Fourier series for $f(x) = \begin{cases} L, \ if\ 0 \le x < \frac{L}{2} \\ x - L, if\ \ \frac{L}{2} \le x \le L \end{cases}$. **(10 Marks)**

c). Given that $f(x,y) = xy + ye^x$ find $f_{xx}$ and $f_{yy}$. **(5 Marks)**

d). Express $\frac{1}{2}\oint_C 4xdy - 3ydx$ in the form $\iint_R f(x,y)\,dxdy$. **(4 Marks)**

**QUESTION THREE (20 MKS)**

a). Find the Taylor 4th polynomial for $f(x) = \cos 2x$ at $x = \pi$. **(6 Marks)**

b). Find the Laplace transform of $f(t) = \frac{1}{4}(e^{2t} + e^{-2t})$. **(4 Marks)**

c). Find the Fourier cosine series of $f(x) = \frac{L}{2}$ on $-L \le x \le L$. **(4 Marks)**

d). Using cylindrical coordinates evaluate $\iint_s z^2(x^2+y^2)\,dxdy$ where $S$ is the surface bounded by the circle $x^2+y^2=4$ on $z=4$. **(6 Marks)**

**QUESTION FOUR (20 MKS)**

a). Find the Curl and Gradient of $\boldsymbol{r} = z\cos y\,\boldsymbol{i} + \sin xy\,\boldsymbol{j} - z\boldsymbol{k}$ at (1,0,1) **(6 Marks)**

b). Evaluate $\int_0^1 \int_0^x \int_0^y (4z^3x - y^3x)dz\,dy\,dx$ **(7 Marks)**

c). Find the Laplace transform of $f(t) = \sin at$ **(7 Marks)**

**QUESTION FIVE (20 MKS)**

a). Using Laplace transform, solve the initial value problem for $t > 0$

$$y''(t) - 10y'(t) + 9y(t) = 5t;\quad y(0) = -1,\ y'(0) = 2$$ **(14 Marks)**

b). Find the total work done in moving a particle in a force field given by $\boldsymbol{F} = 3xy\boldsymbol{i} - 5z\boldsymbol{j} + 10x\boldsymbol{k}$ along the curve $x = t^2+1, y = 2t^2, z = t^3$ from $t=1$ to $t=2$. **(6 Marks)**