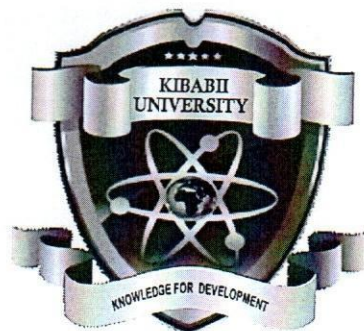


05

5



*(Knowledge for Development)*

**KIBABII UNIVERSITY**

**UNIVERSITY EXAMINATIONS**

**2021/2022 ACADEMIC YEAR**

**THIRD YEAR SPECIAL/SUPPLEMENTARY EXAMINATION**

**FOR THE DEGREE OF BACHELOR OF SCIENCE**

**(RENEWABLE ENERGY)**

**COURSE CODE: MAT 351**

**COURSE TITLE: COURSE TITLE: ENGINEERING MATHEMATICS III**

**DATE: 18/11/2022**

**TIME: 2 PM -4 PM**

---

### **INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

### QUESTION ONE (30marks)

(a). Define the following terms

(i). Laplace transform of a function  $f(t)$  (2 Mark)

(ii). Step function a function (1 Mark)

(iii). Laplace operator (2 Mark)

(iv). Periodic function (1 Mark)

b). Find the **div** (**curl**  $\mathbf{v}$ ) given that  $\mathbf{v} = x^2\mathbf{i} - 2xy^2\mathbf{j} + 2z\mathbf{k}$ . (5 Marks)

c). Find the inverse Laplace transform of  $\frac{4s^2+12}{s(s^2+4)}$  (5 Marks)

d). Find the directional derivative of  $\phi = x^2y + zx$  at  $(1,1,1)$  in the direction of  $\mathbf{u} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ . (5 Marks)

e). (i). Show that the following limit does not exist

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x+y}{x^2+y^2} \quad (4 \text{ Marks})$$

f). Find the Fourier sine series for  $f(x) = x^2$  on  $0 \leq x \leq L$  (5 Marks)

### QUESTION TWO (20 MKS)

a). When are two functions said to be orthogonal? (1 Mark)

b). Find the Fourier series for  $f(x) = \begin{cases} L, & \text{if } 0 \leq x < \frac{L}{2} \\ x - L, & \text{if } \frac{L}{2} \leq x \leq L \end{cases}$  (10 Marks)

c). Given that  $f(x, y) = xy + ye^x$  find  $f_{xx}$  and  $f_{yy}$ . (5 Marks)

d). Express  $\frac{1}{2} \oint_C 4xdy - 3ydx$  in the form  $\iint_R f(x, y) dx dy$ . (4 Marks)

### QUESTION THREE (20 MKS)

a). Find the Taylor 4<sup>th</sup> polynomial for  $f(x) = \cos 2x$  at  $x = \pi$ . (6 Marks)

b). Find the Laplace transform of  $f(t) = \frac{1}{4}(e^{2t} + e^{-2t})$ . (4 Marks)

c). Find the Fourier cosine series of  $f(x) = \frac{L}{2}$  on  $-L \leq x \leq L$ . (4 Marks)

- 79
- d). Using cylindrical coordinates evaluate  $\iint_S z^2(x^2 + y^2) dx dy$  where  $S$  is the surface bounded by the circle  $x^2 + y^2 = 4$  on  $z = 4$ . (6 Marks)

**QUESTION FOUR (20 MKS)**

- a). Find the Curl and Gradient of  $\mathbf{r} = z \cos y \mathbf{i} + \sin xy \mathbf{j} - z \mathbf{k}$  at  $(1,0,1)$  (6 Marks)
- b). Evaluate  $\int_0^1 \int_0^x \int_0^y (4z^3 x - y^3 x) dz dy dx$  (7 Marks)
- c). Find the Laplace transform of  $f(t) = \sin at$  (7 Marks)

**QUESTION FIVE (20 MKS)**

- a). Using Laplace transform, solve the initial value problem for  $t > 0$   
$$y''(t) - 10y'(t) + 9y(t) = 5t; \quad y(0) = -1, \quad y'(0) = 2 \quad (14 \text{ Marks})$$
- b). Find the total work done in moving a particle in a force field given by  $\mathbf{F} = 3xy\mathbf{i} - 5z\mathbf{j} + 10x\mathbf{k}$  along the curve  $x = t^2 + 1, y = 2t^2, z = t^3$  from  $t = 1$  to  $t = 2$ . (6 Marks)