



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2021 /2022 ACADEMIC YEAR
THIRD YEAR SECOND SEMESTER
SUPPLEMENTARY EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN
MATHEMATICS

COURSE CODE: MAP 321

COURSE TITLE: REAL ANALYSIS III

DATE: 23/11/2022

TIME: 11:00 AM – 1:00 PM

Answer question ONE and any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (20 MARKS)

- a) Define the following terms
- Sequence
 - Riemann-Integrable function
 - Fourier series
- b) Show that if $\lim_{n \rightarrow \infty} a_n \neq 0$ or if $\lim_{n \rightarrow \infty} a_n$ fails to exist, then $\sum_{n=1}^{\infty} a_n$ diverges
- c) Assume α increases monotonically and $\alpha \in R$ on $[a, b]$, let f be a bounded real function on $[a, b]$. show that $f \in R(\alpha)$ if and only if $f\alpha \in R$. In that case, $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$

QUESTION TWO (20 MARKS)

- a) Show that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges
- b) Show that if $\sum_{n=1}^{\infty} \frac{1}{n!}$ Converges
- c) Show that if $A = \sum_{n=1}^{\infty} a_n$ and $B = \sum_{n=1}^{\infty} b_n$ both exist and are finite, then
- $\sum_{n=1}^{\infty} (a_n + b_n) = A + B$ is monotonic, that is $\mu(E) \leq \mu(F)$ whenever E and F are sets in R such that $E \subset F$
 - $\sum_{n=1}^{\infty} k a_n = k \sum_{n=1}^{\infty} a_n = kA$ ($k \in \mathbb{R}$)

QUESTION THREE (20 MARKS)

- a) State the five properties of the integral
- b) Show that if p^* is a refinement of p , then $L(p, f, \alpha) \leq L(p^*, f, \alpha')$ and $U(p^*, f, \alpha') \leq U(p, f, \alpha)$
- c) Suppose $c_n \geq 0$ for $1, 2, 3, \dots$ $\sum c_n$ converges, $\{s_n\}$ is a sequence of distinct points on (a, b) and $\alpha(x) = \sum_{n=1}^{\infty} c_n I(x - s_n)$. Let f be continuous on $[a, b]$ then show $\int_a^b f d\alpha = \sum_{n=1}^{\infty} c_n f(s_n)$

QUESTION FOUR (20 MARKS)

- a) Given the function $y=f(x)$ obtained by introducing the continuous variable x in place of the discrete variable in the n th term of the positive series $\sum_{n=1}^{\infty} a_n$ be a decreasing function of x for $x \geq 1$, show that the series and the integral $\int_1^{\infty} f(x)dx$ both converge or both diverge
- b) Show that if p is a real constant, the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p < 1$

QUESTION FIVE (20 MARKS)

- a) Suppose φ is strictly increasing continuous function that maps an interval $[a, b]$, suppose α is monotonically increasing on $[a, b]$ and $f \in R(\alpha)$ on $[a, b]$. Define β and g on $[A, B]$ by $\beta(y) = \alpha(\varphi(y))$ and $g(y) = f(\varphi(y))$ show that $\int_A^B d\beta = \int_a^b d\alpha$
- b) Show that $f \in R(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition p such that $U(p, f, \alpha) - L(p, f, \alpha) < \varepsilon$
- c) State the comparison test for series of nonnegative terms