(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FORTH YEAR SECOND SEMESTER

SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAP 422**

COURSE TITLE: MEASURE THEORY AND **INTEGRATION**

DATE: 17/11/2022 **TIME:** 8:00 AM - 10:00 AM

Answer question ONE and any other TWO Questions

TIME: 2 Hours

QUESTION ONE (20 MARKS)

a) Define the following terms

i. Set function

ii. Measure

iii. Finite set function

iv. Positive set function

b) Show that if E is a set in R, and $E = \bigcup_1^r [a_i b_i) = \bigcup_1^s [c_j d_j)$ are two representations of E, then $\sum_1^r (b_i - a_i) = \sum_1^s (d_j - c_j)$

c) State the unique Extension Theorem (UET)

d) Show that if $\alpha_i \uparrow \alpha$ and $\beta_i \uparrow \beta$ then $\alpha_i + \beta_i \uparrow \alpha + \beta$

QUESTION TWO (20 MARKS)

a) Given R is a ring of subsets of a set X, show that the monotone ring generated by R coincides with the ring generated by R; $M(R) = G(R)$

b) Given μ is a measure on a ring R, show that

i. μ is monotonic, that is $\mu(E) \leq \mu(F)$ whenever E and F are sets in R such that $E \subset F$

ii. μ is conditionally subtractive that is $\mu(E - F) \leq \mu(F) - \mu(E)$ whenever E and F are sets in R such that $E \subset F$ and $\mu(E)$ is finite

iii. μ is finitely additive that is if $E_1, \ldots, E_n$ are mutually disjoint sets in R, then $\mu(\bigcup_1^n E_n) = \sum_1^n \mu(E_n)$

iv. μ is countably additive that is, if E_k is a sequence of mutually disjoint sets in R such that $\bigcup_1^\infty E_k$ is in R, then $\mu(\bigcup_1^\infty E_k) = \sum_1^\infty \mu(E_k)$ in the sense that the LUB of the (increasing) sequence of partial sums $\sum_1^\infty \mu(E_k)$ is equal to $\mu(\bigcup_1^\infty E_k)$

QUESTION THREE (20 MARKS)

a) Define the following terms

i. Union

ii. Intersection

iii. Difference

iv. Symmetric difference

v. Family of elements

b) Show that if v is an outer measure, the class M of v-measurable sets is a ring

c) State the Lemma on Monotone classes (LCM)

QUESTION FOUR (20 MARKS)

a) Explain the following terms

   i. $f = g$ almost everywhere

   ii. $f \leq g$ almost everywhere

   iii. f is constant almost everywhere

b) Show that if f_n is a sequence of integrable functions such that $f_n \geq 0$ a.e. and $\lim inf \int f_n \, d\mu < \infty$, then there exists an integrable function f such that $f = \lim\inf f_n \; a.e.$ and one has $\int f \, d\mu \leq \lim\inf \int f_n d\mu$

QUESTION FIVE (20 MARKS)

a) Show that (μ_i) is an increasingly directed family of measures on a ring R and μ is the set function on R defined by the formula $\mu(E) = LUB \; \mu_i(E),$ then μ is a measure on R. Notation $\mu = LUB \; \mu_i$

b) Show that if f, g, h are extended real valued functions on X, then

   i. $f = f$ almost everywhere

   ii. $If \; f = g$ almost everywhere then $g = f$ almost everywhere

   iii. $if \; f = g$ a.e. and $g = h$ a.e. then $f = h$ a.e.