(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 422/MAA 421**

COURSE TITLE: PDE II

DATE: 18/11/2022 **TIME**: 11:00AM - 01:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE COMPULSORY (30 MARKS)

(a) Define the following terms;

i. Reducible linear differential operator.

ii. Linear non-homogeneous partial differential equation. (4 marks)

(b) Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = x$ (6 marks)

(c) Determine the complete solution of

$$\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x + 2y)$$ (6 marks)

(d) Using method of separation of variables, solve

$\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-5x}$ (6 marks)

(e) Using method of characteristics solve the semi-linear partial differential equation.

$4\frac{\partial u}{\partial y} - 2\frac{\partial u}{\partial x} + 5u = 0$ (4 marks)

(f) Given the following PDE;

$$y\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial y^2} = 0$$

i. Determine the region in the $xy - plane$ where it is elliptic.

ii. Determine its characteristic curves. (4 marks)

QUESTION TWO (20 MARKS)

(a) Solve the following wave equation using the method of separation of variables

$\frac{\partial^2 y}{\partial t^2} = 25\frac{\partial^2 y}{\partial x^2}$

with boundary conditions

$y = 0$, when $x = 0$ and $x = 2$

and initial conditions

$\frac{\partial y}{\partial t} = 0$, and $y(x, 0) = 6\sin\left(\frac{\pi x}{2}\right) - 3\sin(\pi x)$ when $t = 0$, $0 < x < 2$ (14 marks)

(b) Assume $n = 1$ and $u(x,t) = v\left(\frac{x}{\sqrt{t}}\right)$, show that $u_t = u_{xx}$ if and only if

$v''(z) + \frac{v'(z)}{z} = 0 \quad (z > 0)$ where the prime indicates differentiation with respect to z and

$z = \frac{x}{\sqrt{t}}$ (6 marks)

QUESTION THREE (20 MARKS)

(a) Differentiate between Laplace's equation and Poisson's equation. (2 marks)

(b) Find the temperature function $u(x,t)$ on an insulated metallic rod of length L which is governed by $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions $u(0,t) = 0, u(L,t) = 0$and $u(L,t) = \frac{100\,x}{L}$

(13 marks)

(c) Solve $(D^2 + D')(D + 4D' - 6)z = 0$ (5 marks)

QUESTION FOUR (20 MARKS)

(a) Classify the given PDE below and determine its characteristic curves

$$y^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + x^2 \frac{\partial^2 u}{\partial y^2} = y^3 \frac{\partial u}{\partial x} + x^4 \frac{\partial u}{\partial y}$$ (3 marks)

(b) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} - 2z = x^3 y^2$ (11 marks)

(c) Solve the wave equation by D'Alembert's method

$\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$ where C is a constant (6 marks)

QUESTION FIVE (20 MARKS)

(a) Formulate the following 2-Dimensional Laplace's Equation in polar coordinates and solve it (give its general solution). $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (14 marks)

(b) Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = x$ (6 marks)